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Complete rate equation modelling of quantum cascade lasers for the analysis of temperature effects

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HIGHLIGHTS

- Thermal behaviour of GaAs-based quantum cascade lasers has been analysed.
- A complete rate equation model considering all the scattering events is presented.
- Analytical expressions for the laser characteristics parameters have been derived.
- Model is experimentally validated both for higher and lower temperatures.
- A small signal direct intensity modulation response analysis is performed.

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ABSTRACT

The effect of temperature on the dynamics of a GaAs-based quantum cascade laser (QCL) is analysed using a complete rate equation model. The analytical expressions for the threshold current density and the output power are derived using the model and the thermal behaviour of these parameters is examined. A better conformity of the threshold current density with experiment at higher temperatures is achieved. The effect of temperature on the 3 dB optical bandwidth is further investigated using the same model. A comparative analysis of the model is performed with the recently reported rate equation models. It is observed that the 3 dB optical bandwidth falls more rapidly at higher operating temperatures that highlight the effects of leakage and backscattering processes present in the device.

1. Introduction

Quantum cascade lasers (QCLs) [1,2] are finding a great deal of attention in many applications due to their unique mid- to far-infrared wavelength operations. QCLs are unipolar devices where the photon emission takes place due to the intersubband transition of carriers (electrons or holes) between the lasing states in the active region (AR) of a multi-quantum-well system and the lasing wavelength can be controlled by engineering the AR well thicknesses. The population inversion is achieved by selective injection of carriers into the upper lasing level through resonant tunnelling and a faster depopulation of the lower lasing level by the emission of longitudinal optical (LO) phonons. The nonradiative relaxation times which determine the QCL characteristics depend strongly on the temperature of operation and limit the operating temperature range of the device. Room temperature operation of a mid-infrared QCL [3] and maximum ~200 K operations [4] without application of a magnetic field of a terahertz QCL are achieved experimentally.

The major nonradiative scattering processes, responsible for determining the states lifetimes and the broadening of absorption linewidth are LO phonon scattering, ionized impurity (IIMP) scattering, interface roughness (IFR) scattering [5,6] and electron-electron (e-e) scattering [7]. The e-e scattering rate is important for states with a high level of carrier population and significantly small energy separation [8]. QCLs operating at longer wavelengths (more than 8 μm) are strongly influenced by IFR scattering [9]. Besides, in mid-infrared QCLs, IFR scattering serves as the dominant loss mechanism of carriers from the lasing states to the upper excited states at higher temperatures when the energy separations between the lasing states and the upper excited states are considerably small [10]. The QCL systems with energy separation

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between states more than LO phonon energy and with small doping densities are dominated by LO phonon scattering. In this work, we have only considered the LO phonon scattering as it is the dominating scattering mechanism in our case and other scattering processes are neglected.

At higher operating temperatures, the backfilling of the lasing states due to backscattering of carriers from the lower energy states in the AR becomes significant due to thermally activated nonradiative scattering processes which change the laser characteristics remarkably. The leakage of the carriers at higher temperatures by different leakage channels plays a very important role in determining the laser characteristics. The different leakage channels include leakage of carriers from the upper laser level to the continuum states and the states within the AR. For higher conduction band offset QCLs such as InP-based devices the carrier leakage is primarily a shunt leakage current within the AR [11–13] and for lower conduction band offset QCLs such as GaAs-based devices the leakage is primarily to the continuum. The backfilling of the lower lasing state by thermal electrons from the injector states is negligible as the relaxation process from the upper injector states to the ground injector state is assumed to be very fast compared to the relaxations in the AR and a large spatial separation between the injector ground state and lower lasing state. The variation of the full width at half maximum (FWHM) of the electroluminescence (EL) spectrum with temperature due to the variation of the nonradiative scattering processes with temperature modify the temperature characteristics of a QCL.

In the present work, first the finite difference method (FDM) is used to solve the time independent effective mass Schrödinger equation considering temperature dependent conduction band potentials. Then the energies and the wave functions obtained using FDM are applied to the scattering rate equation to calculate the lifetimes of the AR states. A three level rate equation model is presented considering all the scattering processes involved in the QCL. The backfilling of the AR states owing to the backscattering of the carriers at higher temperatures by LO phonon reabsorption from the lower energy states is considered in the rate equation model. The leakage of carriers from the upper lasing state by thermal excitation of electrons to the continuum states and the relaxation of carriers to the lower AR states are also incorporated in the model. Next, the model is used to derive the expressions for the threshold current density and the output power analytically. The change in the level population of the lasing states due to spontaneous emission is also included in the model. The effect of hot electrons close to room temperature and the effect of interface roughness on the carrier lifetimes and EL linewidth are neglected in our model. Our result shows a better agreement with the experimental result at higher temperature than the approximation model reported by Hamadou et al. [14]. The effect of backscattering to the threshold current density and the output power is shown. Finally, a small signal direct intensity modulation analysis is performed by using the rate equations presented in this paper and numerical result for the 3 dB optical bandwidth is obtained and the effect of phonon assisted backscattering is examined. A comparison of our model with a recently reported rate equation model for the small signal analysis is performed. The results highlight the importance of incorporating phonon backscattering, particularly at higher temperatures.

This paper is arranged in 4 consecutive sections. First section gives a brief idea about the work presented in the paper. Some theoretical aspects, used for the development of the model and deriving the results are discussed in Section 2. Section 3 provides numerical results obtained from the model and gives insight into them. Lastly in Section 4 all the findings are highlighted and the work is concluded.

2. Theoretical consideration and problem formulation

To develop a theoretical model for the thermal characterization of a QCL, we have used a QCL structure reported in Ref. [15]. The AR of the QCL is a three well structure of $Al_{0.45}Ga_{0.55}As/GaAs$ followed by an injector region. The widths of the layers (in nm) in the AR are 4.6, 1.9, 1.1, 5.4, 1.1, 4.8 and 2.8. The first layer is the injection barrier. The electron energies and the corresponding wave functions for the system have been calculated by solving the Schrödinger equation considering the temperature variation of the conduction band offset by means of Varshni relation using FDM and shown in Fig. 1.

2.1. Rate equation modelling

A three level rate equation model for a QCL system to describe the carrier statistics in each level in the AR can be given as [16]

$$\frac{dN_3}{dt} = WL\frac{J}{e} - \frac{N_3}{\tau_{3e}} + \frac{N_2}{\tau_{23}} + \frac{N_1}{\tau_{13}} - G(N_3 - N_2)N_{ph}, \tag{1.a}$$

$$\frac{dN_2}{dt} = \left(\frac{1}{\tau_{32}} + \frac{1}{\tau_{sp}}\right)N_3 - \left(\frac{1}{\tau_{21}} + \frac{1}{\tau_{23}}\right)N_2 + \frac{N_1}{\tau_{12}} + G(N_3 - N_2)N_{ph}, \tag{1.b}$$

$$\frac{dN_1}{dt} = \frac{N_3}{\tau_{31}} + \frac{N_2}{\tau_{21}} - \left(\frac{1}{\tau_{12}} + \frac{1}{\tau_{13}} + \frac{1}{\tau_{out}}\right)N_1, \tag{1.c}$$

$$\frac{dN_{ph}}{dt} = NG(N_3 - N_2)N_{ph} - \frac{N_{ph}}{\tau_p}, \tag{1.d}$$

where N_{ph} is the population number for the cavity photons and N_3, N_2 and N_1 are the population numbers for the carriers of the upper lasing level, lower lasing level and the lowest energy level which is used for faster depopulation of the lower lasing level, respectively. J is the current density injected to the upper lasing level and $G = (\Gamma c'\sigma_{32}/V)$ is the coefficient of the optical gain where $V = NWLL_p$ is the AR volume of the QCL structure with N number of cascades of length L_p along the growth axis. W and L are the sidewise dimensions of the QCL cavity. $c' = c/n_{eff}$, where c is the light speed in vacuum and n_{eff} is the effective refractive index. The term σ_{32} signifies the stimulated emission cross section and is written a $\sigma_{32} = \left\{\left(4\pi e^2 Z_{32}^2\right)/(\varepsilon_0 n_{eff}\lambda(2\gamma_{32}))\right\}$. The lifetime of photon (τ_p) inside the QCL cavity depends mainly on the waveguide loss of

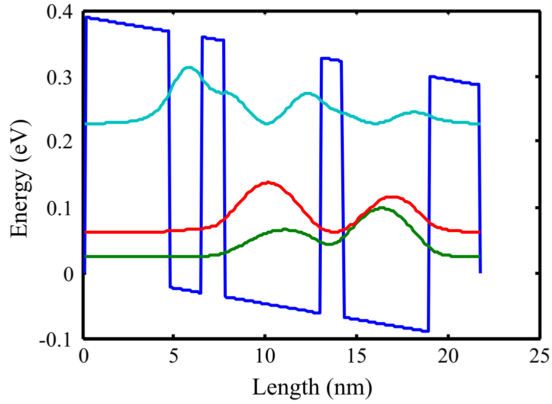

Fig. 1. Energy band diagram of the QCL AR under electric field of 48 kV/cm. Electron energies and the corresponding modulo squared wave functions are shown.