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Closed formulae for the wave number of Sommerfeld's wire line with a large radius

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Abstract

Simple closed formulae are derived for the number of a harmonic wave traveling along a straight conducting wire, whose diameter may be larger than that the wavelength in vacuum. Those formulae are validated against a numerical method which solves the propagation equation exactly. Two examples show that the frequency-range of application of the proposed formulae extends into the infrared.

Keywords

surface waves, transmission lines, Sommerfeld wire, Goubau line, continued fractions, Bessel functions, THz, infrared, Drude's law

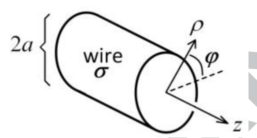

Fig. 1. Sommerfeld's wire line.

1 Introduction

The transmission line of Sommerfeld [1], also known as Goubau line [2], is sketched in Fig.1. It is a straight conducting round wire with radius *a*, which stretches through vacuum along the *z*-axis. Owing to its low loss and low dispersion, it is a favourite waveguide for operation at terahertz (THz) frequencies [3]. It spares one from using some of the bulky optical parts that are customary in THz systems, while lending itself to the propagation of subpicosecond pulses [4].

Let k_0 and γ_z be the respective wave numbers of vacuum and of that wire's fundamental TM_0 mode of propagation, at frequency f. Until recently one has used an algorithm [5,6] to solve the equation for γ_z, except when $k_0 a \ll 1$ [7]. That equation involves Bessel functions of a complex variable, whose computation is difficult and time-consuming. The closed formulae for γ_z, that Jaisson published last year [8] are for the case where $k_0 a$ equals at most one wavelength. That ballpark limit was set by the approximations that Jaisson applied to the Bessel functions. In many cases, it is not restrictive, from the user's point of view. But when $f > 1\ \mathrm{THz}$, going into the infrared range [9], the requirement for mechanical strength requires that the wire be electrically thicker, that is $k_0 a > \pi$. For lack of more general approximations for the said Bessel functions, one must treat the case of the so-called thick wire separately, as is done in this paper.

In Section 2, one approximates the propagation equation of the TM_0 mode, to a cubic equation. The latter is easily solved, for it involves no Bessel function. One determines the frequency-range where the formulae thus obtained for γ_z are valid, in Section 3. In Section 4, one compares the values obtained for γ_z by means of those formulae, to the solution of the exact propagation equation, in two examples.

2 Approximate propagation equation

In this Section one recalls the propagation equation of the TM_0 mode of Sommerfeld's wire line. Then one approximates the Bessel functions involved, whereby one obtains an equation which is solvable by hand.

Let $\omega = 2\pi f$, and let factor $\exp(\mathrm{j}\omega t)$ express the harmonic fields' dependence on time t, as a convention $(\mathrm{j}\cdot\mathrm{j} = -1)$. One considers a wire made out of a non-magnetic metal with conductivity σ and relative permittivity [9,10]

$$\varepsilon_\mathrm{r} = 1 + \sigma / \mathrm{j}\omega\varepsilon_0 \tag{1}$$