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Time-domain viscoelastic constitutive model based on concurrent fitting of frequency-domain characteristics

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ABSTRACT

A numerical procedure for constructing the multiaxial viscoelastic model for polymeric packaging material over a wide range of temperature is presented. By using the proposed best-fitting procedure, experimentally measured frequency-domain Young's and shear storage moduli are used to calculate the time-domain bulk and shear relaxation moduli which describe the three-dimensional constitutive behavior of a viscoelastic solid. The numerical procedure incorporates restrictions that ensure that the derived time-domain material function is physics compatible. The proposed procedure was applied to construct the viscoelastic constitutive models of epoxy molding compounds (EMCs), and compared to results obtained by using approximate-formula based direct conversion procedure. It was shown that, without using the proposed procedure, the directly calculated time-dependent Poisson's ratio oscillates significantly in the rubbery regime and is physically inadmissible. To validate the constitutive model constructed by using the proposed procedure, a numerical finite element model that incorporates the viscoelastic constitutive model of the EMC was applied to simulate warpage of an overmolded package under the solder reflow process and compared to experimental shadow Moiré measurements.

1. Introduction

In organic microelectronic assemblies the viscoelastic nature of polymeric packaging materials plays an important role in the overall deformations and stresses because the process temperatures are typically around or higher than the glass transition temperatures (T_g) of the polymeric constituents [1]. Evaluations of the influences of these time-dependent behaviors on the thermomechanical characteristics of the assemblies can be performed by using numerical finite element procedures. In these numerical models, viscoelastic behaviors of the polymeric materials are described by using time-dependent material functions. Within the context of linear viscoelasticity, any two of the relaxation (or creep) functions corresponding to the uniaxial, shear, bulk and Poisson's responses are sufficient to define the three-dimensional constitutive behavior of the viscoelastic solid, and the remaining material functions can be obtained from the two selected functions.

Experimental measurements of the viscoelastic behaviors are typically performed either under quasi-static relaxation (or creep) conditions or under time harmonic oscillating conditions. Aside from the standard quasi-static uniaxial or shear tests, tools such as dynamic mechanical analysis (DMA) instrument under cyclic tensile or bending conditions [2], rheometer under cyclic torsion condition [3], and high pressure dilatometer under compressive creep condition [4] are often used for the viscoelasticity characterization. Among the various tools used for measuring viscoelastic properties, the DMA instrument and rheometer are the most widely adopted because the test durations required in these time harmonic oscillating setups are much shorter than those required in the quasi-static creep or relaxation tests. The viscoelastic material functions obtained from these oscillation tests are typically in the forms of Young's/shear storage and loss moduli as functions of oscillating frequency and temperature. However, in most numerical finite element procedures used for mechanical simulations, the viscoelastic formulations are based on the shear and bulk relaxation functions. As a consequence, additional post-processes are required to convert the Young's and shear storage moduli in the frequency domain to the storage and shear relaxation moduli in the time domain. The post-process would involve converting the frequency-domain viscoelastic functions to the corresponding time-domain functions, and calculating the viscoelastic bulk relaxation modulus from the uniaxial and shear responses. The viscoelastic Poisson's ratio, while is not required for the finite element simulation, should also be calculated in the post-process to assess the accuracy of the multiaxial viscoelastic constitutive behavior described by the corresponding bulk and shear relaxation moduli.

Interconversions of the linear viscoelastic material functions between their values in the frequency and time domains can be achieved by using approximate analytical formulas [5,6] or numerical procedures [7,8]. In general these procedures were shown to give relatively good estimations. On the other hand, the calculation of the bulk and the Poisson's ratio functions are more complicated. It was highlighted by Tschoegl et al. [9] that the smallest specimen-to-specimen property

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variation and differences in the environment variables would result in significant variation or even physically contradicting Poisson's ratio values, and it is required to measure the two independent viscoelastic functions simultaneously on the same specimen in a single experiment to minimize this issue. While the protocol of simultaneously measuring two material functions are essential for the accuracy of the viscoelastic Poisson's ratio, it is very difficult to accomplish in practice, and the reported data are scarce [9]. As an alternative to this rigorous experimental protocol, a numerical best-fitting procedure with restriction on Poisson's ratio's acceptability is proposed to post-process the separately measured frequency-domain Young's and shear storage moduli for determining a set of time-domain bulk and shear relaxation moduli that would meet the requirements for physically admissible multiaxial viscoelastic constitutive model.

In this paper multiaxial viscoelastic constitutive models obtained by using the proposed moduli interconversion procedure are presented for EMCs of two different filler percentages. Experimental characterizations for the frequency dependent Young's and shear storage moduli under various temperatures were first performed by using a DMA instrument and a rheometer, respectively. Mastercurves of the experimental Young's and shear storage moduli were constructed based on the time-temperature-superposition principle. A numerical program based on the proposed viscoelastic function interconversion procedure was then applied to convert the Young's and shear storage moduli to the corresponding time-domain viscoelastic material functions. The mathematical models of the bulk and shear relaxation moduli in the forms of Prony series were implemented in finite element models for simulating the warpage of electronic package under surface mount solder reflow condition, and compared to shadow Moiré experimental measurements to evaluate the accuracy of the viscoelastic model.

2. Multiaxial viscoelastic model

The multiaxial constitutive behavior of an isotropic linear-viscoelastic material can be expressed in a convolution integral form given by

$$\sigma_{ij}(t) = \delta_{ij}\int_{0^-}^{t}\kappa(t-\tau)\frac{d\varepsilon_{kk}(\tau)}{d\tau}d\tau + \int_{0^-}^{t}2\mu(t-\tau)\frac{de_{ij}(\tau)}{d\tau}d\tau,\quad i,j=1,2,3, \tag{1}$$

where σ_{ij} is the stress, ε_{kk} is the sum of normal strains ($\varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$), e_{ij} is the deviatoric strain, δ_{ij} is the Kronecker delta, and κ and μ are the bulk and shear relaxation moduli, respectively. Under uniaxial loading condition, the multiaxial constitutive Eq. (1) reduces to

$$\sigma_{11}(t) = \int_{0^-}^{t}E(t-\tau)\frac{d\varepsilon_{11}(\tau)}{d\tau}d\tau, \tag{2}$$

where E is the Young's relaxation modulus. When the viscoelastic material is subjected to uniaxial time harmonic strain excitation such as in the DMA, the steady-state stress response can be written as

$$\sigma_{11}(t) = \left[E'(\omega) + iE''(\omega)\right]\varepsilon_0 e^{i\omega t}, \tag{3}$$

where $\varepsilon_0 e^{i\omega t}$ denotes the strain excitation, $E'(\omega)$ and $E''(\omega)$ are the Young's storage and loss moduli, respectively, and are given by

$$E'(\omega) = E_\infty + \omega\int_0^\infty [E(t)-E_\infty]\sin(\omega t)dt, \tag{4}$$

$$E''(\omega) = \omega\int_0^\infty [E(t)-E_\infty]\cos(\omega t)dt, \tag{5}$$

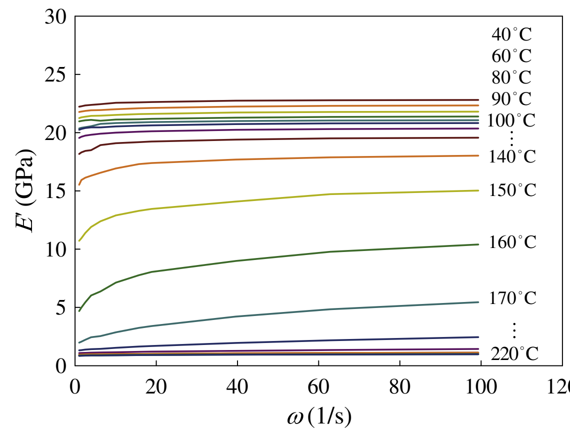

Fig. 1. Young's storage modulus of Compound A.

where E_∞ is the value of E as $t \to \infty$. The inverse relationship of (4) can be expressed as [10]

$$E(t) = \frac{2}{\pi}\int_0^\infty E'(\omega)\frac{\sin\omega t}{\omega}d\omega. \tag{6}$$

For the case of shear loading such as the time-harmonic shear strain excitation used in torsional DMA, the shear stress-shear strain and the related time-frequency domain moduli relationships can be obtained by replacing the normal stress, normal strain, and Young's modulus terms in Eqs. (2)–(6) with the shear stress, shear strain, and shear modulus terms, respectively. The relationship between shear relaxation and storage moduli can be therefore expressed as

$$\mu(t) = \frac{2}{\pi}\int_0^\infty \mu'(\omega)\frac{\sin\omega t}{\omega}d\omega. \tag{7}$$

2.1. Direct interconversion

Because the viscoelastic stress–strain relationships such as Eqs. (1) and (2) are in the forms of convolution integrals, relationships between the material functions cannot be expressed in simple equations. Interconversions of these materials functions including Young's relaxation modulus, viscoelastic Poisson's ratio, bulk relaxation modulus and shear relaxation modulus are typically conducted in the Laplace transform domain, in which the relationships between the material functions are in the forms of algebraic equations. Consequently, a straight-forward approach for determining the time-domain bulk relaxation modulus and viscoelastic Poisson's ratio from the frequency-

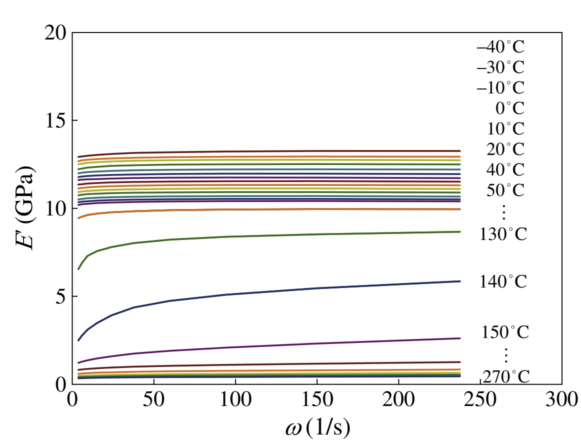

Fig. 2. Young's storage modulus of Compound B.