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Research articles

Analytical solution of concentric two-pole Halbach cylinders as a preliminary design tool for magnetic refrigeration systems

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ABSTRACT

This work presents a parametric analysis of the performance of nested permanent magnet Halbach cylinders intended for applications in magnetic refrigeration and heat pumping. An analytical model for the magnetic field generated by the cylinders is used to systematically investigate the influence of their geometric parameters. The proposed configuration generates two poles in the air gap between the cylinders, where active magnetic regenerators are positioned for conversion of magnetic work into cooling capacity or heat power. A sample geometry based on previous designs of magnetic refrigerators is investigated, and the results show that the magnetic field in the air gap oscillates between 0 to approximately 1 T, forming a rectified cosine profile along the circumference of the gap. Calculations of the energy density of the magnets indicate the need to operate at a low energy (particular the inner cylinder) in order to generate a magnetic profile suitable for a magnetic cooler. In practice, these low-energy regions of the magnet can be potentially replaced by soft ferromagnetic material. A parametric analysis of the air gap height has been performed, showing that there are optimal values which maximize the magnet efficiency parameter Λ_{cool}. Some combinations of cylinder radii resulted in magnetic field changes that were too small for practical purposes. No demagnetization of the cylinders has been found for the range of parameters considered.

1. Introduction

Since the pioneering work of [11], who built the first apparatus to achieve room-temperature refrigeration using the magnetocaloric effect (MCE), several magnetic refrigeration laboratory prototypes have been described in the literature [30,14,2,22,1,15,27,26]. However, none of those designs have been capable of producing values of cooling capacity $\dot{Q}_C$, system temperature span ΔT_{span} (difference between hot and cold sources temperatures), and coefficient of performance COP compatible with commercially available cooling and heat pumping technologies.

The magnetic circuit is a critical component in a magnetic cooler, for it is responsible for providing the necessary magnetic field variations, which drive the MCE. The magnetic circuit is the most expensive and voluminous component of a magnetic refrigerator [5]. Most devices published to this date (including the ones previously cited) use some variant of the permanent magnet Halbach cylinder [18] as the magnetic circuit. In this axisymmetric structure, the direction of magnetization varies continuously along the circumference. Kitanovski et al. [23] state that the concept of a Halbach cylinder rotating around a configuration of active magnetic regenerator (AMR) beds is one of the best design configurations for magnetic refrigerators.

To understand the behavior of Halbach cylinders, Bjørk et al. [9] developed analytical solutions of the magnetic field in the inner and outer regions of a single cylinder. Also, in the same work, the problem of two concentric cylinders (with an air gap in-between) was considered to calculate the force and the torque to rotate the inner piece, without considering the geometric parameters in the calculation of the magnitude of the magnetic field. In a subsequent work, Bjørk et al. [7] improved the design of four-pole Halbach concentric cylinders by performing numerical simulations and proposing an algorithm to replace parts of the permanent magnet volume with soft ferromagnetic material (e.g. iron) to increase the amplitude of the magnetic field variations. An extension of this geometry optimization procedure, applied to a six-pole Halbach cylinder, was performed by You et al. [31]. Other approaches to the optimization of Halbach cylinders have been studied in the literature. An alternative methodology of replacing regions of the magnetic circuit with soft ferromagnetic material, with an emphasis on guiding the flux lines through the desired paths, was presented by Lorenz and Kevlishvili [25]. Insinga et al.

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[19] showed an efficient algorithm to find the globally optimal shapes of iron and magnet segments (instead of testing the effect of pre-defined shapes) subjected to volume constraints.

In more recent works, topology optimization has been applied to the design of magnetic circuits. Lee et al. [24] investigated the design of Halbach arrays, where the shapes of the segments are not changed during the optimization procedure, but only the direction of remanence of each fixed-form segment. Bjørk et al. [6] used a different methodology in which each point in the design space could be assigned either to a permanent magnet or an iron region, for both the problems of single and concentric Halbach cylinders, and showed that the efficiency of a magnetic circuit can be greatly improved.

Other works have focused on different approaches to the design of Halbach arrays. Bouchekara et al. [10] described the design of a magnetic circuit composed of rotating bar-shaped magnets, arranged in line with different directions of magnetization (following the Halbach pattern); the relative rotation between them create an oscillating magnetic field in the regions above and below the line of magnets, where regenerators can be placed. Trevizoli et al. [28] presented a design of two nested Halbach cyclinders, with the air gap in the common core of both cylinders, using only permanent magnet material (with no iron parts) capable of generating high magnetic fields using a small footprint for the magnetic circuit. Using a similar configuration, Arnold et al. [2] improved the magnetic circuit from Tura and Rowe [30] with three nested Halbach cylinders and showed that the magnetic field variations are increased.

This paper advances analytical solutions for two-pole nested Halbach cylinders by extending the single-cylinder solution of Bjørk et al. [9]. The influence of the geometric parameters of the cylinders on the performance of the magnetic circuit is further investigated. A greater emphasis is placed on the air gap volume between the concentric cylinders, as it represents a fundamental design trade-off in a magnetic refrigerator/heat pump: for a fixed total volume of the magnetic system, a larger gap volume yields a larger volume of magnetocaloric material in the regenerator, which is beneficial for the cooling system overall performance, but results in a smaller volume of the magnetic circuit, which tends to decrease the overall performance due to lower magnetic field variations. This clearly demonstrates the existence of optimal magnetic circuit dimensions with respect to magnetic refrigeration.

2. Mathematical model

2.1. Problem geometry

The problem geometry is shown in Fig. 1, where only the first quadrant is shown for simplicity. The two permanent magnet Halbach cylinders are represented by regions II and IV; region I is a shaft needed to rotate the inner magnet; region III is the magnetic gap where AMR beds containing the magnetocaloric material are placed; region V is the external environment and region VI is a fictitious region with infinite magnetic permeability needed for model closure. Thus $R_e \to \infty$ will be assumed from now on. The position vector in a cylindrical coordinate system is defined as $\boldsymbol{r} = r\hat{\boldsymbol{e}}_r + \phi\hat{\boldsymbol{e}}_\phi + z\hat{\boldsymbol{e}}_z$. In the infinitely long cylinder array considered here, the magnetic flux density field $\boldsymbol{B}$ and the magnetic field $\boldsymbol{H}$ are essentially two-dimensional, with null components in the axial direction, z.

2.2. Constitutive relations

Each region k in Fig. 1 is assumed to be uniformly filled with a linear and isotropic magnetic material which is characterized by a constant relative permeability $\mu_{r,k}$ and a magnetic remanence $\boldsymbol{B}_{\mathrm{rem},k}$. The constitutive relation for each region is:

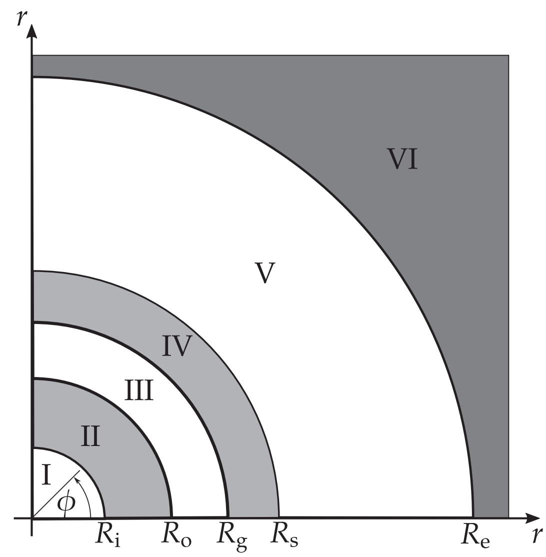

Fig. 1. Problem geometry considered in the analytical solutions of the magnetic field generated by two concentric Halbach cylinders. Regions are as follows: I: shaft; II: inner Halbach cylinder; III: magnetic gap; IV: outer Halbach cylinder; V: external environment; VI: hypothetical region with infinite magnetic permeability.

$$\boldsymbol{B}_k = \mu_0 \mu_{r,k} \boldsymbol{H}_k + \boldsymbol{B}_{\mathrm{rem},k} \quad (1)$$

where the magnetic remanence $\boldsymbol{B}_{\mathrm{rem},k}$ is non-null only for the permanent magnet regions II and IV.

Permanent magnets (also called hard ferromagnetic materials) have an easy axis of magnetization [12,21], and their magnetic response is dominated by interactions in the direction parallel to their magnetic remanence [20]. Let the index $\parallel$ represent the components of the vector fields in that direction. Taking the dot product of Eq. (1) with $\boldsymbol{B}_{\mathrm{rem},k}$ (and omitting the index k for simplicity) gives:

$$B_\parallel = \mu_0 \mu_r H_\parallel + B_{\mathrm{rem}} \quad (2)$$

Thus, a hysteresis curve $B_\parallel(H_\parallel)$ is generated due to an applied field $H_\parallel$. Permanent magnets operate on the second quadrant of this curve, the so-called *demagnetization curve* [13], which is shown in Fig. 2. The *operating point* of a permanent magnet is usually represented by the point $(H_\parallel, B_\parallel)$. When the operating point shifts to the first quadrant ($H_\parallel > 0$), the permanent magnet no longer acts as a source of magnetic flux, so it can be replaced by *soft* ferromagnetic material with a high relative permeability to help guide the flux lines and intensify the magnetic field [5].

In the vicinity of the *intrinsic coercivity* $-H_{c,i}$, the hard ferromagnetic material suffers demagnetization [13,28,21] and can no longer perform as a permanent magnet. When $H_\parallel < -H_{c,i}$, the operating point shifts to the third quadrant [21], Eq. (2) is no longer valid, and the problem becomes non-linear, making it difficult to obtain analytical solutions. Hence, in this work, it is assumed that the permanent magnet always operates in the linear portion of Fig. 2; Section 4.2 will show some situations where this assumption might no longer hold.

2.3. Halbach cylinders

For two-dimensional permanent magnet Halbach cylinders, the remanence can be expressed as [9]:

$$\boldsymbol{B}_{\mathrm{rem},k}(r,\phi) = B_{\mathrm{rem},k}\left(\cos(p_k\phi)\,\hat{\boldsymbol{e}}_r + \sin(p_k\phi)\,\hat{\boldsymbol{e}}_\phi\right) \quad (3)$$