Journal of Magnetism and Magnetic Materials ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Dynamic analysis on magnetic fluid interface validated by physical laws

Yo Mizuta

Division of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan

ARTICLE INFO

Article history:
Received 30 June 2016
Received in revised form
12 August 2016
Accepted 5 September 2016

Keywords:
Magnetic fluid
Magnetic field
Free surface
Transition
Nonlinear interaction
Numerical analysis

ABSTRACT

Numerical analyses of magnetic fluid especially for fast phenomena such as the transition among interface profiles require rigorous as well as efficient method under arbitrary interface profiles and applied magnetic field distributions. Preceded by the magnetic analysis for this purpose, the present research has attempted to investigate interface dynamic phenomena. As an example of these phenomena, this paper shows the wavenumber spectrum of the interface profile and the sum of interface stresses changing in time, since the change of the balance among the interface stresses causing the transition can be observed conveniently. As time advances, wavenumber components increase due to the nonlinear interaction of waves. It is further argued that such analyses should be validated by the law of conservation of energy, the relation between the interface energy density and the interface stress, and the magnetic laws.

1. Introduction

On the interface of magnetic fluid, unique phenomena such as the transition among interface profiles are observed. The interface which was initially flat abruptly changes to the profile with a regular pattern, when the intensity of an applied magnetic field is increased as close as to the critical value. Just after the magnetic fluid was invented, linear analyses [1,2] or weakly nonlinear analyses [3–5] were used for this phenomenon, where quantities on the interface are approximated by finite power series of the interface elevation which is assumed to be small. During the transition process, various nonlinear interactions of waves are considered to work, but the above analyses investigated just the onset of the process.

It is an interesting problem to follow the process of pattern formation as time advances. However, analyses for such problem require rigorous as well as efficient methods under arbitrary interface profiles and applied magnetic field distributions. Preceded by the magnetic analysis for general use developed for this purpose (Section 4) [6,7], the present research has attempted the analysis on interface dynamic phenomena, such as the bifurcation of the interface stability [7,8] and the wavenumber spectra of interface quantities changing in time (Section 3 of this paper).

Numerical analyses should be accompanied by the confirmation that obtained fluid and magnetic quantities satisfy physical laws sufficiently. This is especially important for fast phenomena such as the transition process. As shown in Section 2, the interface elevation ζ is obtained by integrating in time the equation for interface motion, which includes the sum of interface stresses S. Then, the validation problem is divided into two: First is the time integration which can be verified by the law of conservation of energy; second is the correctness of S itself which is verified by the relation with the interface energy density (interface energy per unit area in Flat Space), as discussed in Section 5. In addition, with respect to magnetic quantities, the relation between the magnetic interface energy density and the Maxwell stress should be verified together with Ampére's law and Gauss's law (Section 6).

2. Equation for interface motion

When we analyze free surface phenomena of incompressible, irrotational and inviscid magnetic fluid with all nonlinear effects but with no limitations on the interface profile, we use the following equation for interface motion which is derived by using the kinematic and dynamic conditions on the interface for the tangential component of the fluid equation of motion [7]:

E-mail address: yomizuta@eng.hokudai.ac.jp

http://dx.doi.org/10.1016/j.jmmm.2016.09.030
0304-8853/

$$\rho\frac{\partial\varphi}{\partial t} + S = 0, \quad S \equiv D + G + C + T + p_0, \tag{1}$$

$$T = -\,[1/\mu_j]\{\mu_1\mu_2(h_X^2 + h_Y^2) + b_Z^2\}/2, \tag{2}$$

where ρ, φ, D, G, C, T, and p_0 are the fluid density, velocity potential, dynamic pressure, gravity potential, surface tension, magnetic stress difference and atmospheric pressure, respectively. In (1), φ is obtained from the vertical component of the fluid velocity v_z and the interface elevation ζ as $\varphi = \int_{-\infty}^{\zeta} dz v_z$. Furthermore, T represents the action from the magnetic field to the fluid, where μ_j denotes the permeability of the fluid ($j = 1$) or the vacuum ($j = 2$), $[\cdots]$ the difference of the value across the interface $(2 - 1)$. The tangential magnetic field $h_{X,Y}$ and the normal magnetic flux b_Z can be obtained rigorously as well as efficiently under arbitrary interface profiles and applied magnetic field distributions, as shown in Section 4 [6,7]. In addition, we omit D and p_0 in S hereafter supposing that the interface moves slowly enough, and the atmospheric pressure is homogeneous.

The interface elevation $\zeta(\boldsymbol{R})$ and the sum of interface stresses $S(\boldsymbol{R})$ as functions of the interface coordinate parameter $\boldsymbol{R} = (X, Y)$ are expressed as the superposition of periodic functions with the wavenumber $\boldsymbol{k}$:

$$\zeta(\boldsymbol{R}) = \sum_{\boldsymbol{k}} \zeta_{\boldsymbol{k}}(\boldsymbol{R}), \quad S(\boldsymbol{R}) = \sum_{\boldsymbol{k}} S_{\boldsymbol{k}}(\boldsymbol{R}). \tag{3}$$

Then, (1) is rewritten as

$$0 = \sum_{\boldsymbol{k}} \left\{ \frac{\partial}{\partial t}\left(\rho\frac{\partial\zeta_{\boldsymbol{k}}}{\partial t}\right) - \frac{1}{k}\nabla_2^2 S_{\boldsymbol{k}} \right\}, \tag{4}$$

where $\nabla_2 = (\partial/\partial X, \partial/\partial Y)$ is the partial derivative in the tangential direction, and $k = |\boldsymbol{k}|$.

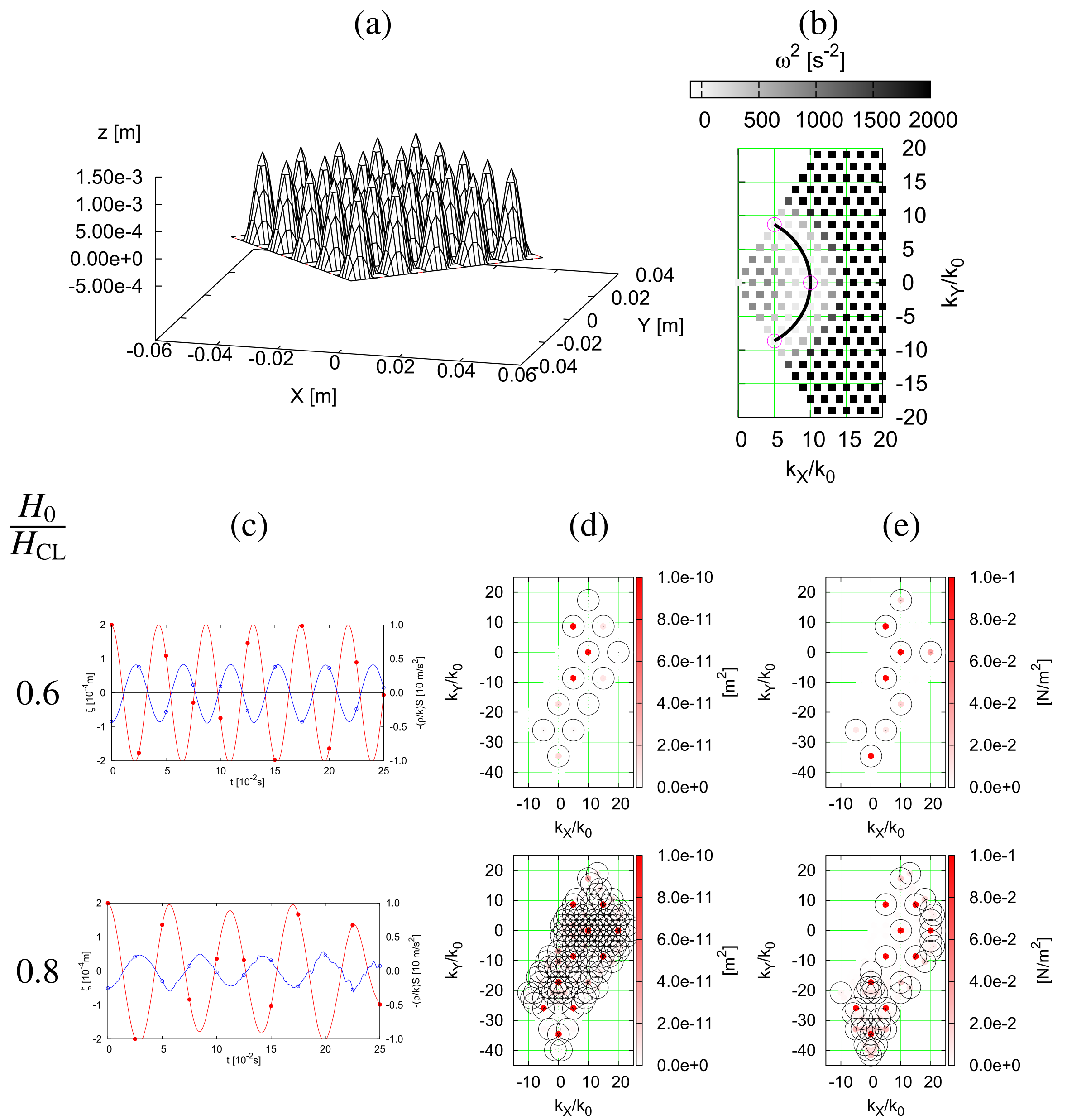

Fig. 1. Interface profile of hexagonal lattice in (a) real space and (b) wavenumber space. (c) Temporal change in basic wavenumber component of interface elevation $\tilde{\zeta}_{\boldsymbol{k}}$ (red) and force caused by sum of interface stresses $-(k/\rho)\tilde{S}_{\boldsymbol{k}}$ (blue). (d), (e) Wavenumber spectrum of interface elevation $|\tilde{\zeta}_{\boldsymbol{k}}|^2$ and sum of interface stresses $|\tilde{S}_{\boldsymbol{k}}|^2$ at time 20.0×10^{-2} s (specific permeability of magnetic fluid $\mu_1/\mu_0 = 1.2$, and initial amplitude of basic wavenumber component of interface elevationis $\zeta_0 = 0.2$ mm). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)