Author's Accepted Manuscript

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PII: S0304-8853(16)31571-2
DOI: http://dx.doi.org/10.1016/j.jmmm.2016.07.057
Reference: MAGMA61669

To appear in: *Journal of Magnetism and Magnetic Materials*

Received date: 30 June 2016
Accepted date: 26 July 2016

Cite this article as: N.M. Zubarev and O.V. Zubareva, Deformation of the free surface of a conducting fluid in the magnetic field of current-carrying linear conductors, *Journal of Magnetism and Magnetic Materials* http://dx.doi.org/10.1016/j.jmmm.2016.07.057

Deformation of the free surface of a conducting fluid in the magnetic field of current-carrying linear conductors

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Abstract – **The magnetic shaping problem is studied for the situation where a cylindrical column of a perfectly conducting fluid is deformed by the magnetic field of a system of linear current-carrying conductors. Equilibrium is achieved due to the balance of capillary and magnetic pressures. Two two-parametric families of exact solutions of the problem are obtained with the help of conformal mapping technique. In accordance with them, the column essentially deforms in the cross section up to its disintegration.**

Keywords – ***magnetic shaping, free surface, conducting fluid, linear current-carrying conductors, exact solutions.***

1. Introduction

The free surface of a liquid possessing magnetic properties is deformed under the influence of the applied magnetic field. A large number of papers are devoted to investigations of possible equilibrium configurations of the ferromagnetic fluid boundary in an external magnetic field (see, for example, [1-5] and references therein). Of considerable interest is also the behavior of conducting fluids under the action of magnetic field [6-9]. It can be assumed approximately that a high-frequency magnetic field does not penetrate into the fluid with sufficiently high conductivity. In this case, the lines of force of the field are tangent to the boundary outside the liquid. The appearing normal magnetic forces result in the deformation of the surface. In their turn, the time-averaged magnetic forces can be counterbalanced by capillary forces at the deformed boundary. Note that the problem of finding the respective equilibrium configurations is equivalent to the problem concerning the boundary shape of a perfectly conducting liquid placed into a constant magnetic field not penetrating into the medium.

Finding possible equilibrium surface configurations is a rather sophisticated problem. The reason is that the force balance condition at the liquid boundary is essentially nonlinear and nonlocal (in order to determine the magnetic pressure, the magnetic field distribution should be found in the whole space). For the plane geometry of the problem, using the conformal mapping method, the initial problem with an unknown boundary can be reduced to the problem with known boundary, which represents a circle or a straight line. This approach was used to analyze the shapes of liquid cylindrical conductors in a high-frequency magnetic field [6,9]. In Ref. [10], we have obtained exact particular solutions for the shape of the initially plane surface of a perfectly conducting fluid subjected to a nonuniform magnetic field of a horizontal linear current-carrying conductor. Finally, in Ref. [11], two one-parameter families of exact solutions have been obtained for the equilibrium configurations of the free surface of a liquid jet deformed by a magnetic field that is generated by two thin conductors that are parallel to the jet and bear oppositely directed currents.

In the present work, we will demonstrate that the approach suggested in [11] can be extended to the case of a system of $2n$ parallel conductors situated around the liquid column. The practical purpose of such investigations is to analyze the feasibility of controlling the shape of the molten metal boundary by the applied magnetic field [12].

2. Basic equations

Consider a cylindrical column of a perfectly conducting incompressible liquid (liquid metal). Its surface is deformed by the magnetic field of $2n$ infinitely long straight thin conductors carrying oppositely directed currents $\pm I$. The conductors are placed circumferentially at distance L from the column axis. At $I=0$, when the magnetic field is absent, the column has a shape of a circular cylinder with radius R_0.

Let us introduce rectangular coordinate system $\{x,y,z\}$ so that the z axis coincides with the column's axis. We assume that the column deforms only in the plane of its cross section $\{x,y\}$, i.e., all quantities depend only on two variables, x and y. The problem is invariant under rotation around the column axis at the angle of $\Theta=\pi/n$. Then, it can be considered in the wedge-shaped domain with the angle Θ, including a single current-carrying conductor.

For the plane symmetric case, the distribution of the magnetic field $\mathbf{B}$ is defined by the scalar function ψ:

$$\mathbf{B}=\left\{\psi_y,-\psi_x,0\right\}.$$

This function has the meaning of z component of the vector potential of the magnetic field $\mathbf{A}=\left\{0,0,\psi(x,y)\right\}$, where $\nabla\times\mathbf{A}=\mathbf{B}$. Note that the condition $\psi=\text{const}$ sets the force lines of the magnetic field.