Journal of Magnetism and Magnetic Materials 441 (2017) 62–68

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Research articles

A continuous analysis for the antiferromagnetic Heisenberg model on the checkerboard lattice

A.R. Moura *, A.R. Pereira

Departamento de Física, Universidade Federal de Viçosa, 36570-900 Viçosa, Minas Gerais, Brazil

ARTICLE INFO

Article history:
Received 27 June 2016
Received in revised form 8 May 2017
Accepted 12 May 2017
Available online 17 May 2017

Keywords:
Checkerboard lattice
Antiferromagnetic Heisenberg model
Critical spin
O(3) Non-linear Sigma Model
Skyrmions

ABSTRACT

In this paper we investigate the antiferromagnetic Heisenberg model on the checkerboard lattice by the $O(3)$ Non-linear Sigma Model (*NLSM*). The checkerboard lattice is distinguished from the antiferromagnetic square lattice (with coupling constant J) by the presence of diagonal crossing (with coupling constant J') in half of the sites. This lattice model is the direct analogous of the three-dimensional pyrochlore lattice in a two-dimensional surface. Many effects of the three-dimensional model version, as the Quantum Order-by-Disorder, have been recently described also in the checkerboard lattice. Here we have developed the continuous version of the Heisenberg model on the checkerboard lattice and applied Renormalization Group together other techniques to analyze the both cases $J < J'$ and $J > J'$. We have therefore determined the effects of the crossing interaction J' on the phase transitions. In addition, skyrmion solutions and their possible influences on these transitions were also considered.

1. Introduction

Even after many years, frustrated models, specially in magnetism, have surprised physicists with fascinating new phenomena. Since the 1980 decade, one-dimensional liquid spin is known from theoretical and experimental works but one can not surely assert the same about two-dimensional models. Yet, nowadays there is an uncertainty about the possibility of a real disordered spin state at zero temperature and, notwithstanding the many candidate models to present such a state, there is not a conclusive study about the two-dimensional spin liquids. It is believed that frustration may be a key ingredient to create states without any kind of spin order even at zero temperature. Frustration is associated with the inability of the system to reach a singular state that minimizes the energy and so, in frustrated systems, the ground state is highly degenerated. Curiously, thermal and quantum fluctuations raise the degeneracy, allowing the system to become ordered again. This symmetry broken driven by fluctuations is the so-called Order-by-Disorder (*ObD*) effect (or Order-by-Quantum-Disorder when quantum perturbations are responsible for the ground state selection). The checkerboard lattice is a model where the *ObD* effect is predicted by many different approaches [1–6]. In summary, if the spin-waves are non-interacting, then the antiferromagnetic checkerboard lattice is able to develop a spin liquid state but, when the spin-waves interaction is present in the system, the ground state is ordered at zero temperature. This ordering is due to spin field fluctuations.

In this paper we have applied the continuous approach to the antiferromagnetic Heisenberg Hamiltonian to the checkerboard lattice. Here, we represent the checkerboard model by two interconnected sublattices with order parameter a (smaller distance between two neighbor spins) as shown in Fig. (1). The Hamiltonian is given by

$$H = J\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j + J'\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j, \tag{1}$$

where the first sum is over nearest neighbor spins on different sublattices (with coupling constant $J > 0$) and the other one is over neighbor spins on the same sublattice (with coupling constant $J' > 0$). Instead using J and J', we adopt the ratio $\eta = J'/J$ along the text.

The checkerboard structure is composed by a square lattice with diagonal crossing in half of the square cells; note that the yellow squares in Fig. (1) is different from the ones in $J - J'$ model, in which the crossing interactions are present in all cells. The checkerboard lattice is really appealing due to its similarities with the planar pyrochlore lattice (the two-dimensional projection of the tetrahedral pyrochlore lattice). In fact, in the fully frustrated limit, when $\eta = 1$, both models have very similar properties. The

* Corresponding author.
E-mail addresses: antoniormoura@ufv.br (A.R. Moura), apereira@ufv.br (A.R. Pereira).

http://dx.doi.org/10.1016/j.jmmm.2017.05.036
0304-8853/

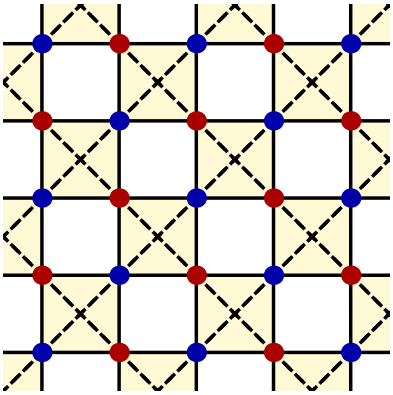

Fig. 1. The checkerboard lattice is composed by two interconnected square lattices, each one represented by a different color (only in the online version). The solid lines represent the J interaction while the dotted ones are the J' interaction. The yellow squares represent the cells considered in the work. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

ground state, for instance, is a gapped phase with no symmetry breaking.

In the limit of weak crossing interaction, $\eta \ll 1$, we recover the two-dimensional antiferromagnetic square lattice. At zero temperature the ground state of this system is the collinear Néel long-range order with gapless excitations, different from the $\eta = 1$ case. Therefore, it is expected a phase transition which separates the gapless phase from the gapped phase at a critical ratio $\eta_c = J'_c/J < 1$. Since magnetic compounds with small spin are more susceptible to quantum fluctuations, it is also expected a smaller critical point η_c for spin-1/2. If we consider spin-wave interactions, then only models with spin-1/2 could support states with no magnetization. The cases with spin-1 (and higher spin values) always present some continuous symmetry breaking at zero temperature. For spin-1/2, Sindzingre et al. [7] have used Exact Diagonalization to show the closure of the gap at $\eta_c = 0.65$, while Khatami and Rigol [8] have demonstrated the disappearance of the *AF* order at low temperatures when $\eta = 0.75$ (considering spin-1/2). These results are close to the semiclassical approaches (obtained from the linear spin-wave theory) which predict a gapless ordered phase for $\eta \leqslant 0.75$ ($\eta_c \approx 0.93$ for $S = 1$, at this level of approximation). However, Canals [9] has shown that the interaction between spin-waves increases considerably the critical point to $\eta_c = 0.98$.

Above the critical point η_c, the checkerboard model presents a gapped Valence Bond Crystal (*VBC*) with long-range quadrumer order [2,10–13]. On the other hand, in the limit $J = 0$, there is an ensemble of non-coupled one-dimensional antiferromagnetic spin chains. The *AF* spin chains are authentic spin liquid states with gapless excitations (deconfined spinons) for semi-integer spins and gapped excitations for integer values of spin. The integer spin chain gap was firstly predicted by Haldane using the $O(3)$ Non-linear Sigma Model (*NLSM*), the continuous representation of the Heisenberg model. The absence of the Haldane gap for the semi-integer spin chain is purely due to topological effects which, in general, are absent in higher dimensional models. The region $J' > J$ is not yet well documented and it is one of the aims of the present work.

The continuous approximation allows us to determine the properties of the excitations in both limits $J > J'$ and $J < J'$. Here, we have calculated the critical spin S_c which separates the gapless phase with Long-Range-Order (*LRO*) from the gapped one, where the order-parameter decays exponentially. The gapped spectrum excitation is an indicative of a possible liquid spin phase.

2. The continuous model

2.1. First case $J > J'$

To develop the continuous model we have properly replaced the sum over neighbors sites in Hamiltonian (1) by a sum over cells of a diagonal square centered lattice, represented here by Γ. The cells considered here are represented in Fig. (1) by yellow squares. All the spin interactions are included in the yellow cells and the expressions obtained from the new approach have exactly the same physics as the original Hamiltonian. We denote the position of a cell by $\vec{r}$ and the spins on the corners by $\vec{r}_{ij} = \vec{r} + \Delta\vec{r}_{ij}$ where $\Delta\vec{r}_{ij} = a(i,j)$ and $i,j = 0,1$.

In order to get the continuous approach, we have used the staggered configuration with wave vector $\vec{Q} = (\pi/a, \pi/a)$ as the ground state (coherent with the limit $J' < J$). Since each cell contains four spins, we need four independent vector fields [14] to correctly keep the freedom degrees. For a cell in the position $\vec{r}$, the four corners spins are represented by

$$\vec{S}(\vec{r}_{00}) = S\vec{m} + aS(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) \tag{2a}$$

$$\vec{S}(\vec{r}_{10}) = -S\vec{m} + aS(\vec{l}_1 - \vec{l}_2 + \vec{l}_3) \tag{2b}$$

$$\vec{S}(\vec{r}_{01}) = -S\vec{m} + aS(-\vec{l}_1 + \vec{l}_2 + \vec{l}_3) \tag{2c}$$

$$\vec{S}(\vec{r}_{11}) = S\vec{m} + aS(-\vec{l}_1 - \vec{l}_2 + \vec{l}_3) \tag{2d}$$

The field $\vec{m}$ defines the staggered field $\vec{n} = (-1)^{i+j}\vec{m}$, while the fields $\vec{l}$ are three small deviations from $\vec{n}$. All fields $\vec{m}$ and $\vec{l}$ are evaluated at position $\vec{r}$. The lattice parameter a is introduced to ensure only first order approximation in the series expansion. The constraint $\vec{S}^2 = S^2$ is recovered if the fields obey $m^2 + a^2S^2(l_1^2 + l_2^2 + l_3^2) \approx m^2 = 1$ and $\vec{m}\cdot\vec{l}_i + a\vec{l}_j\cdot\vec{l}_k = 0$, where i,j,k are different indices.

The fields $\vec{l}$ represent the fast modes spin fluctuations and they will be integrated in order to obtain an effective model. As usual, the partition function is obtained using the spin coherent states which, in the imaginary time, provides

$$Z = \int \mathcal{D}\vec{S}\,\delta(\vec{S}^2 - S^2)e^{-\mathcal{S}}, \tag{3}$$

with the action $\mathcal{S} = \sum iS\omega[\vec{S}/S] + \int_0^\beta H(\tau)\,d\tau$. The local delta Dirac function assures the $O(3)$ non-linear constraint $\vec{S}^2 = S^2$ for each site i. The measure of the integration is understood as an integration over the slow and fast modes, $\vec{m}$ and $\vec{l}$, respectively, on each site of the lattice. In the first term, $\omega[\vec{S}/S]$ represents the Berry phase (the kinetic term) and the sum is done over all spins or, as we have adopted, over all cells in Γ. The second term in the action (the potential term) is given by the mean value of Hamiltonian (1) in the coherent state basis.

Using the parametrization fields we have obtained the continuous representation of the kinetic term given by

$$\sum_{\text{spins}} iS\omega[\vec{S}/S] = \frac{iS}{2}\sum_{r\in\Gamma}\sum_{ij}\omega[\vec{S}(\vec{r} + \Delta\vec{r}_{ij})/S]$$
$$= \frac{iS}{a}\int d^2r \int_0^\beta d\tau\vec{l}_3\cdot\left(\frac{\partial\vec{m}}{\partial\tau}\times\vec{m}\right). \tag{4}$$

Here we have adopted only first order terms in the expansion of the Berry phase functional. On the other hand, Chaudhury and Paul [15,16] have included higher order terms in the expansion to describe also the medium-wavelength limit of the spin-wave in the two-dimensional quantum Heisenberg antiferromagnet. However the medium wave-length analysis is out the scope of the