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Reciprocal relations for nonlinear multipole in inhomogeneous magnetic field

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ABSTRACT

In this paper the reciprocal relations for the matrix coefficients of the resistance of nonlinear multipole located in an inhomogeneous magnetic field was considered. It is shown that the nonlinear resistance matrix can be represented as the sum of two matrices. The coefficients of the first matrix depend on both the current flowing through the multipole and the external inhomogeneous magnetic field. First matrix is responsible for nonlinear effects. The second matrix coefficients depend only on the induction of the external inhomogeneous magnetic field and are responsible for the Hall effect and the offset resistance. We obtain reciprocal relations for these matrices and experimentally show that the classical reciprocal relations are valid for the second matrix within the limits of experimental accuracy.

1. Introduction

Studying of linear and nonlinear transport processes in magnetic and non-magnetic materials is a well-developed research field in the physics of magnetic phenomena [1,2]. The interest in theoretical and experimental studying of the influence of magnetic field on reciprocal relations for kinetic coefficients is mostly due to the widespread use of non-reciprocal passive components in magnetic microwave devices [3]. In recent years, there has been an increased interest in magnetoelectronic devices which utilize both the charge and the spin of the electron, since these devices are expected to provide unprecedented functionalities for energy efficient spin-based information processing. The examples of such spintronic devices are giant magnetoresistance spin valve and magnetic tunnel junction [4]. Room-temperature reversible spin Hall effect, Onsager reciprocal relations between spin and charge currents [5], and spin-transfer torque [6] open up the possibility of switching one of the magnetic layers by means of spin-polarized current, thus paving the way to compact devices for random access memory applications.

Magnetic materials used in modern spin-wave electronics have essentially nonlinear susceptibility and conductivity and are influenced by inhomogeneous magnetic field such as surface magnetostatic waves. Analytical and numerical simulations of uniaxial ferromagnets, used in magnetoelectronic devices shows that they exhibit nonlinear susceptibility and very distinct non-reciprocity [8]. However, classical reciprocal relations for the kinetic coefficients [7] are valid in linear case for homogeneous magnetic field. Hence, the applicability of reciprocal relations for analysis, calculation and optimization of signal characteristics of magnetoelectronic devices is of topical interest.

In recent years, much effort was devoted to obtaining reciprocal relations in the particular cases of nonlinear and inhomogeneous systems such as two-dimensional systems with nonlinear conductivity [9]. However, the influence of external magnetic field and the internal magnetization of material on reciprocal relations for nonlinear systems has not yet been considered.

Common statistical methods for mathematical analysis of reciprocal relations are inapplicable in nonlinear case [10]. Thus, obtaining general reciprocal relations for a nonlinear system in inhomogeneous magnetic field is not possible without some assumptions about processes taking place in it. Therefore, it is of particular interest to justify such relations for the most general assumptions for measuring physical quantities with high precision without disturbing the processes in the system. A possible example of such a system is a nonlinear multipole with transport processes which are described by the kinetic equation.

2. Theory

Theoretical analysis is divided into several parts. Derivation of the material equation of a nonlinear homogeneous medium in

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the relaxation approximation is considered in Section 2.1. Both current density and electric field intensity inside a solid cannot be measured. Therefore, Section 2.2 concerned with derivation of electric potential distribution for specified currents through external terminals on the basis of the obtained material equation in magnetostatic approximation. In Section 2.3 we obtain reciprocity relations for matrix of nonlinear resistances of a magnetoactive multipole.

2.1. Nonlinear conductivity in magnetic field

Transfer processes in homogeneous and stationary medium in the relaxation approximation are described by the Vlasov kinetic equation [11]:

$$q(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial \mathbf{p}} = -\frac{f-f_0}{\tau}, \tag{1}$$

where $f(\mathbf{p})$ is the distribution function of charge carriers over the momentum $\mathbf{p}$, $f_0(p^2)$ – is the equilibrium distribution function, q is the charge of the carrier, τ is the ensemble mean relaxation time. In the stable state of medium which is isolated from any external influences except electric and magnetic fields, the distribution function of $f(\mathbf{p})$ is uniquely determined by the applied magnetic and electric fields. It means the uniqueness of the solution of Eq. (1).

Let us proceed with the new variables. Assuming that the external magnetic and electric fields are static and homogeneous, and that they are nonzero and not codirectional, let us denote

$$Y_1 = \mathbf{pE},\; y2 = \mathbf{pB},\; y3 = \mathbf{p}[\mathbf{E}\times\mathbf{B}] \tag{2}$$

We then can represent the momentum $\mathbf{p}$ via vectors $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{E}\times\mathbf{B}$:

$$\mathbf{p} = A_1\mathbf{E} + A_2\mathbf{B} + A_3[\mathbf{E}\times\mathbf{B}], \tag{3}$$

where A_i are scalar coefficients. To determine these coefficients, we multiply equation (3) by vectors $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{B}\times\mathbf{E}$:

$$\mathbf{p} = \frac{B^2y_1 - \mathbf{EB}y_2}{|\mathbf{E}\times\mathbf{B}|^2}\mathbf{E} + \frac{E^2y_2 - \mathbf{EB}y_1}{|\mathbf{E}\times\mathbf{B}|^2}\mathbf{B} + \frac{y_3}{|\mathbf{E}\times\mathbf{B}|^2}[\mathbf{E}\times\mathbf{B}]. \tag{4}$$

Note that the there is a one-to-one correspondence between transformations (2) and (4) and

$$\frac{\partial f_\alpha}{\partial \mathbf{p}} = \mathbf{E}\frac{\partial F_\alpha}{\partial y_1} + \mathbf{B}\frac{\partial F_\alpha}{\partial y_2} + [\mathbf{E}\times\mathbf{B}]\frac{\partial F_\alpha}{\partial y_3}, \tag{5}$$

where the distribution function $F(y_1, y_2, y_3)$ is obtained from function $f(\mathbf{p})$ by replacing the variables (4):

$$f(\mathbf{p}) = F(y_1 = \mathbf{pE},\; y_2 = \mathbf{pB},\; y_3 = \mathbf{p}[\mathbf{E}\times\mathbf{B}]) \tag{6}$$

Then Eq. (1) is then transformed to:

$$\left(E^2 + \frac{y_3}{m}\right)\frac{\partial F}{\partial y_1} + \mathbf{EB}\frac{\partial F}{\partial y_2} + \left(B^2y_1 - \mathbf{EB}y_2\right)\frac{\partial F}{\partial y_3} = \frac{F-F_0}{q\tau}, \tag{7}$$

where m is the effective mass of the charge carrier.

Let us notice that in the Eq. (2) the variables y_1, y_2 are linearly dependent on $\mathbf{E}$ and $\mathbf{B}$. In Eq. (7), they are considered as independent variables, while the components of momentum $\mathbf{p}$ are expressed through them according to (4). So, when we change the signs of the fields $\mathbf{E}$ and $\mathbf{B}$, the variables y_1, y_2, y_3 in Eq. (7) do not change. In addition, although the components of momentum $\mathbf{p}$ change when the signs of fields of $\mathbf{E}$ and $\mathbf{B}$ change, the Jacobian J of transformation (4) does not change. Indeed, taking into account the Eq. (2) we get:

$$J = \frac{\partial(p_1, p_2, p_3)}{\partial(y_1, y_2, y_3)} = \frac{1}{|\mathbf{E}\times\mathbf{B}|^2}. \tag{8}$$

Let us limit ourselves to consideration of stable states of the medium. Eq. (7) is invariant to simultaneous inversion of the fields $\mathbf{E}$ and $\mathbf{B}$. Thus, the unique solution of (7) is also invariant to the simultaneous inversion of the fields $\mathbf{E}$ and $\mathbf{B}$:

$$F(y_1, y_2, y_3, -\mathbf{E}, -\mathbf{B}) = F(y_1, y_2, y_3, \mathbf{E}, \mathbf{B}) \tag{9}$$

The constitutive equation for stationary, homogeneous and isotropic medium, taking into account Eq. (4), is as follows [11]

$$\mathbf{j} = \int \frac{cq\mathbf{p}}{m} f(\mathbf{p})d^3p = K_1(\mathbf{E},\mathbf{B})\mathbf{E} + K_2(\mathbf{E},\mathbf{B})\mathbf{B} + K_3(\mathbf{E},\mathbf{B})[\mathbf{E}\times\mathbf{B}], \tag{10}$$

where c – charge carrier density,

$$\begin{aligned} K_1(\mathbf{E},\mathbf{B}) &= \tfrac{cq/m}{|\mathbf{E}\times\mathbf{B}|^4}\int\{B^2y_1 - \mathbf{EB}y_2\}F(\mathbf{y})d^3y, \\ K_2(\mathbf{E},\mathbf{B}) &= \tfrac{cq/m}{|\mathbf{E}\times\mathbf{B}|^4}\int\{E^2y_2 - \mathbf{EB}y_1\}F(\mathbf{y})d^3y, \\ K_3(\mathbf{E},\mathbf{B}) &= \tfrac{cq/m}{|\mathbf{E}\times\mathbf{B}|^4}\int y_3F(\mathbf{y})d^3y. \end{aligned} \tag{11}$$

As follows from Eqs. (9) and (11)

$$K_n(\mathbf{E},\mathbf{B}) = K_n(-\mathbf{E},-\mathbf{B}), \quad n = 1,\ 2,\ 3. \tag{12}$$

Note that the coefficients $K_n(\mathbf{E}, \mathbf{B})$ in the form of Eq. (11) are undetermined if $\mathbf{E} = 0$ and $\mathbf{B} = 0$. Let us use an artificial method for analyzing constitutive Eq. (10) in the case of weak fields. Nonlinearity can be considered substantial in the medium when the energy acquired by charge carriers during mean free path l_T is comparable with kT. In this terms, weak electric field means $|\mathbf{E}| << E_T = \varphi_T/l_T$, where $\varphi_T = kT/q$ is the thermal potential of the primary charge carriers. The constitutive equation for homogeneous and isotropic medium in magnetic field has the form of Ohm law in which Lorentz force is added to Coulomb force:

$$\mathbf{j} = cq\nu\left(\mathbf{E} + \frac{1}{cq}[\mathbf{j}\times\mathbf{B}]\right),$$

where ν is the mobility of charge carriers. Solving this equation with respect to the current density vector $\mathbf{j}$, we get

$$\mathbf{j} = \frac{\sigma}{1+|\nu\mathbf{B}|^2}\mathbf{E} + \frac{\sigma\nu^2(\mathbf{EB})}{1+|\nu\mathbf{B}|^2}\mathbf{B} + \frac{\sigma\nu}{1+|\nu\mathbf{B}|^2}[\mathbf{E}\times\mathbf{B}], \tag{13}$$

where $\sigma = cq$ is the conductivity of medium at zero electric and magnetic fields.

Comparing relations (10) and (13), let us write down the constitutive equation for the medium in the form of

$$\begin{aligned} \mathbf{j} = {} & \sigma\{1 + \alpha(\mathbf{E}/E_T, \nu\mathbf{B})\}\mathbf{E} + \sigma\nu E_T\beta(\mathbf{E}/E_T, \nu\mathbf{B})\mathbf{B} \\ & + \sigma\nu\gamma(\mathbf{E}/E_T, \nu\mathbf{B})[\mathbf{E}\times\mathbf{B}]. \end{aligned} \tag{14}$$

Here, $\alpha(\mathbf{E}=0,\ \mathbf{B}=0) = \beta(\mathbf{E}=0,\ \mathbf{B}=0) = 0, \quad \gamma(\mathbf{E}=0,\ \mathbf{B}=0) = 1$, and, according to Eq. (12),

$$\alpha(-\mathbf{E},-\mathbf{B}) = \alpha(\mathbf{E},\mathbf{B}), \quad \beta(-\mathbf{E},-\mathbf{B}) = \beta(\mathbf{E},\mathbf{B}), \quad \gamma(-\mathbf{E},-\mathbf{B}) = \gamma(\mathbf{E},\mathbf{B}). \tag{15}$$

2.2. Magnetostatic approximation

Let us consider a generalized structure of galvanomagnetic element in the form of a three-dimensional multipole D with M contacts (S_m, $m = 1, 2, \ldots, M$). Let the linear sizes of the element be much larger than the mean free path of the charge carriers l_T. Finally, let $\mathbf{B} = \mathbf{B}_e + \mathbf{B}'$ be the total induction of magnetic field in the element, where $\mathbf{B}_e$ is the field created by external sources and $\mathbf{B}'$ is the field which is generated by current flowing through the galvanomagnetic element. The inhomogeneous field $\mathbf{B}'$ inside the element can significantly exceed the external field $\mathbf{B}_e$ at high current densities. Therefore, the charge carriers in the element will be in the substantially inhomogeneous field $\mathbf{B}$. The constitutive