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Modulational instability and localized modes in Heisenberg ferromagnetic chains with single-ion easy-axis anisotropy

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ABSTRACT

A semiclassical theoretical study on the property of the modulational instability of corresponding linear spin-waves and the presence of nonlinear localized excitations in a discrete quantum ferromagnetic spin chain with single-ion easy-axis anisotropy is reported. We consider the Glauber coherent-state representation combined with the Dyson-Maleev transformation for local spin operators as the basic representation of the system, and derive the equation of motion by means of the Ehrenfest theorem. Using a modulational instability analysis of plane waves, we predict the existence regions of bright envelope solitons and intrinsic localized spin-wave modes. Besides, with the help of a semidiscrete multi-scale method, we obtain analytical solutions for the bright envelope soliton and intrinsic localized spin-wave mode. Moreover, we analyze their existence conditions, which agree with the results of modulational instability analysis.

1. Introduction

The Modulational instability is a very fundamental topic in the theory of the nonlinear wave [1–10]. Modulational instabilities are dynamical instabilities, which are depicted by an exponential growth of arbitrarily small fluctuations due to the interplay between dispersive and nonlinear effects. In nonlinear lattice models, discrete modulational instability is of elementary importance for the formation of localized excitations, such as envelope solitons and intrinsic localized modes [11,12]. In the last few decades, the discrete modulational instability of plane waves in one-dimensional nonlinear oscillator lattices has attracted a considerable deal of interest [13]. Kivshar and Peyrard firstly pointed out that the modulational instability is a possible mechanism for the generation of localized states in nonlinear oscillator lattices [14]. And then, in Kivshar's another article, he showed that the bright intrinsic localized mode only emerges in the parameter domains where the oscillator lattice system exhibits modulational instability [15]. If modulational instability does not take place in the system, then a dark-type localized mode may exist. Afterward, Daumont et al. found that modulational instability of plane waves in nonlinear oscillator lattices is the first step towards lattice vibration energy localization. Indeed, the modulational instability is a potential mechanism for generation of nonlinear localized modes [16]. Hence, it is important to clarify the nature of the modulational instability for better understanding of the generation of the envelope soliton and the intrinsic localized mode in nonlinear lattices.

In Heisenberg spin chains, as a result of intrinsically intrinsic nonlinearity of both spin-spin exchange coupling interactions and on-site anisotropies, ferromagnetic lattice systems can provide tractable candidates to study localized modes and modulational instability of plane waves. Up to now, a great deal of attention has been paid to the properties of nonlinear localized modes and modulational instability in ferromagnetic chains. Initially, Lai et al. studied intrinsic localized modes in a one dimensional ferromagnetic chain with first- and second-nearest-neighbor exchange couplings, and found that the modulation instability of the extended band-edge mode gives a possible mechanism for the generation of resonance localized excitations from extended modes [17]. Afterward, Nguenang et al. investigated modulational instability of the nonlinear extended spin waves in a ferromagnetic chain with biquadratic isotropic exchange interaction and easy-plane single-ion anisotropy [18]. By this work, they putted out some new features of some nonlinear localized modes in the discrete ferromagnetic lattice system. Very recently, Kavitha et al. investigated modulational instability and highly localized discrete breather modes in a one-dimensional discrete weak ferromagnetic lattice with on-site easy-axis anisotropy because of crystal field effect. They found that Dzyaloshinsky–Moriya interaction and the on-site anisotropy affect significantly the excitation of intrinsic localized modes, and the canted ferromagnetic chain system with the antisymmetric nature can produce the long-lived localized excitation [19]. And then, they also investigate the modulational instability of plane carrier waves in a one-dimensional ferromagnetic spin lattice with higher-order dispersive octu-

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pole–dipole and dipole–dipole coupling interactions [20]. Their results showed that the introduction of long-range dispersive interactions is a very efficient mechanism for generation long-lived intrinsic localized mode excitations with the growth of high amplitude.

In this paper, we study the modulational instability and nonlinear excitations in a discrete Heisenberg ferromagnetic spin chain with the single-ion easy-axis anisotropy in the semiclassical limit. By making use of Glauber's coherent state method combined with Dyson-Maleev representation for spin operators, we the drive equation of motion of the ferromagnetic lattice system, which is a discrete nonlinear equation. First, the modulational instability of a carrier plane wave in the ferromagnetic lattice is studied analytically. A linear stability analysis is carried out to predict under what conditions nonlinear localized modes can appear. By means of the multiple-scale method combined with semidiscrete approximation, on the other hand, we shall search for localized solutions to the equation of motion. More details will be revealed in the following sections.

2. The model and equation of motion

In general, the anisotropy is very important for the intrinsic localization of spin waves in ferromagnetic chain with nearest-neighbor interactions [17]. Especially, the anisotropy of ferromagnetic chains plays a key role in the existence of intrinsic localized modes [21]. Here, we will focus our considerations on a one-dimensional ferromagnetic chain with single-ion easy-axis anisotropy, in which spin and spin are coupled via nearest-neighbor isotropic exchange coupling interactions. The Hamiltonian operator of the ferromagnetic system can be described as

$$H = -2J\sum_j \vec{S}_j \cdot \vec{S}_{j+1} - D\sum_j (S_j^z)^2, \tag{1}$$

where $\vec{S}_j = (S_j^x, S_j^y, S_j^z)$ represents the local spin operator on lattice site j, $J(>0)$ is the nearst-neighbor exchange interaction constants, and D is the single-ion uniaxial anisotropy parameter. Here, the value of D is positive so that the Z axis is an easy axis. Without loss of generality, one can assume that all local spins align along the Zaxis direction when the ferromagnetic spin lattice system is in the ground state.

For the purpose of obtaining the second quantization form of the spin lattice model Hamiltonian, we may introduce the Dyson-Maleev transformation for local spin operators in all lattice sites [22,23]

$$S_j^z = S - b_j^+ b_j, \tag{2}$$

$$S_j^+ = (2S)^{1/2}\left(1 - \frac{b_j^+ b_j}{2S}\right) b_j, \tag{3}$$

$$S_j^- = (2S)^{1/2} b_j^+, \tag{4}$$

where b_j (b_j^+) is local boson annihilation (creation) operator in site j, S represents the magnitude of spin, and $S_j^{\pm} = S_j^x \pm iS_j^y$. For the sake of convenience, the Planck constant has been taken as unit. With Eq. (2)–(4), one can get a bosonlized Hamiltonian, which is

$$\begin{aligned} H = &-2JS\sum_j (b_{j+1}^+ b_j + b_j^+ b_{j+1} - 2b_j^+ b_j) \\ &+ J\sum_j (b_{j+1}^+ b_j^+ b_j b_j + b_j^+ b_{j+1}^+ b_{j+1} b_{j+1} - 2b_{j+1}^+ b_{j+1} b_j^+ b_j) + D(2S-1) \\ &\sum_j b_j^+ b_j - D\sum_j b_j^+ b_j^+ b_j b_j, \end{aligned} \tag{5}$$

where the ground-state energy of the spin lattice system has been ignored for simplicity. Physically, this treatment is reasonable and common.

For the sake of characterizing the components of quantum state of the spin lattice system, we make use of the Glauber coherent-state $\left|\beta_j\right\rangle$ as a representation of the present lattice system [24]. In this representation, the lattice system quantum state $|\Phi(t)\rangle$ can be taken to be a product of the following form

$$|\Phi(t)\rangle = \prod_j \left|\beta_j\right\rangle, \tag{6}$$

with $\langle\Phi(t)|\Phi(t)\rangle = 1$. In principle, one can apply the Ehrenfest theorem to drive the equation of motion on the expectation value of a operator in the Schrödinger picture. Thus, doing a simple calculation can give the equation of motion for the coherent-state amplitude, which is

$$\begin{aligned} i\frac{d\beta_n}{dt} = &\,\omega_0\beta_n - 2JS(\beta_{n-1} + \beta_{n+1}) + J[|\beta_{n-1}|^2\beta_{n-1} + |\beta_{n+1}|^2\beta_{n+1} \\ &+ (\beta_{n+1}^* + \beta_{n-1}^*)\beta_n\beta_n - 2(|\beta_{n-1}|^2 + |\beta_{n+1}|^2)\beta_n] - 2D|\beta_n|^2\beta_n, \end{aligned} \tag{7}$$

where $\omega_0 = 4JS + D(2S-1)$. In the above equation, we can see clearly that the discreteness and the intrinsic nonlinearity in the present Heisenberg ferromagnetic spin lattice are fully included even though this equation of equation is a classical analog of c-number equation.

3. Modulational instability analysis

In this section, we shall analyze the discrete modulational instability of a constant amplitude solution to the equation of motion (7) under a plane-wave perturbation. Furthermore, the feasibility of the localized modes in the present system is certified by linear stability analysis [25,26]. First, we look for an plane-wave solution of the form

$$\beta_n = \beta_0 e^{i(qn-\omega t)}, \tag{8}$$

where β_0 is a constant amplitude, q denotes the wave number of the plane-wave, and ω corresponds to the frequency of the plane-wave that follows a nonlinear dispersion relation, namely,

$$\omega = \omega_0 - 4\cos qJS + 4J(\cos q - 1)\beta_0^2 - 2D\beta_0^2. \tag{9}$$

Theoretically, a discrete modulational instability of plane-waves in the present magnetic system can be explored through analyzing the stability of the plane-wave amplitude as a function of very small perturbation to linearizing the equation on the envelope function of the plane carrier wave. Hence, we need to seek a solution of the form

$$\beta_n(t) = [\beta_0 + \delta\beta_n(t)]e^{i(qn-\omega t)}. \tag{10}$$

Inserting Eq. (10) into Eq. (7) and retaining only the linear terms on $\delta\beta_n$ and $\delta\beta_n^*$ will yield a linear differential equation, which reads

$$\begin{aligned} i\frac{\partial\delta\beta_n}{\partial t} = &-2J_1S(\delta\beta_{n-1}e^{-iq} + \delta\beta_{n+1}e^{iq} - 2\cos q\delta\beta_n) + 2J_1\beta_0^2(\delta\beta_{n-1}e^{-iq} \\ &+ \delta\beta_{n+1}e^{iq}) + 2J_1\cos q\beta_0^2(\delta\beta_{n-1}^* + \delta\beta_{n+1}^*) - 2J_1\beta_0^2(\delta\beta_{n-1} + \delta\beta_{n+1} \\ &+ \delta\beta_{n-1}^* + \delta\beta_{n+1}^*) - 2D\beta_0^2(\delta\beta_n + \delta\beta_n^*). \end{aligned} \tag{11}$$

In general, the above linear equation has a general solution with the following form

$$\delta\beta_n(t) = \begin{pmatrix}\mu\\ \nu\end{pmatrix} e^{i(Qn-\Omega t)} = \mu\cos[(Qn-\Omega t)] + i\nu\sin[(Qn-\Omega t)], \tag{12}$$

where Q and Ω stand for the wavenumber and the frequency of the perturbation amplitude, respectively. Here, μ and ν are real constants. Substituting Eq. (12) into Eq. (11), then one shall get the following linear equations for the present lattice system

$$\begin{pmatrix} A_{11} - \Omega & A_{12} \\ A_{21} & A_{22} - \Omega \end{pmatrix}\begin{pmatrix}\mu\\ \nu\end{pmatrix} = 0. \tag{13}$$

The condition existence for non-trivial solutions to the above linear equations can be depicted as the following second order equation on the frequency Ω