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Semi-compact skyrmion-like structures

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ABSTRACT

We study three distinct types of planar, spherically symmetric and localized structures, one of them having non-topological behavior and the two others being of topological nature. The non-topological structures have energy density localized in a compact region in the plane, but are unstable against spherically symmetric fluctuations. The topological structures are stable and behave as vortices and skyrmions at larger distances, but they engender interesting compact behavior as one approaches their inner cores. They are semi-compact skyrmion-like spin textures generated from models that allow to control the internal behavior of such topological structures.

1. Introduction

The study of skyrmions first appeared in the context of particle physics [1] and have been extended to various other areas of nonlinear science, in particular as localized structures in a diversity of magnetic materials; see e.g., Refs. [5–7,10,2–4,8,9]. In this work we focus mainly on magnetic skyrmions, dealing with some analytical tools to describe the formation of skyrmions in magnetic materials. The subject concerns several mechanisms working together [11] and describes properties such as topological charge or winding number, vorticity and helicity [12]. In general, the topological charge of a skyrmion may be integer, $Q=\pm1$, half-integer, $Q=\pm1/2$ (half-skymion or vortex) [9,16,13–15,17], and null $Q=0$, the later being non-topological, considered in Ref. [18]. Here one notes that skyrmions with high-topological-number ($Q\geqslant 2$) has also been suggested; see, e.g., the recent work [19].

We will deal with skyrmion-like structures, paying closer attention to compact solutions, as one explains below. Compact skyrmions have been studied before in Ref. [20], in the context of the baby Skyrme model with a specific potential, and also in Ref. [21], to describe magnetic spin textures that appear in magnetic materials. The study of compact skyrmions in magnetic materials is of current interest, since the recent advances on miniaturization is leading us to manipulate magnetic materials at constrained geometries and this may modify the profile and properties of the magnetic spin textures that inhabits the material. An example of this was identified experimentally in Ref. [22], showing how a domain wall may change profile as it is shrank inside a constrained region. For this reason, we think it is worth investigating the possibility of shrinking vortices and skyrmions to a compact region in the plane.

In the current work, we follow the lines of Refs. [23–25] and use scalar fields to model spin textures for which we can control their spin arrangements. The investigation presented in Ref. [21] has found periodic solutions described in terms of the Jacobian elliptic functions, which are then worked out to give rise to compact structures. Here we follow another route, inspired by the recent investigations [26,27] on compact and asymmetric structures of current interest to high energy physics. We then introduce models that provide semi-compact solutions, which are new isolated structures that engender integer and half-integer skyrmion numbers, for which we are able to control the spin textures at their inner core.

The study deals with three distinct models, one describing non-topological solutions, with topological charge $Q=0$, and the two others being able to generate topological structures with integer and half-integer topological charge. In the case of solutions with vanishing topological charge, we show that they are linearly unstable against radial deformations, so they are of limited interest. But we deal with them because one can develop mechanisms to make them stable, although this is out of the scope of the current work. The solutions with integer and half-integer topological charge are linearly stable against radial deformations, so we focus mainly on them, showing how to control their inner core, without modifying their asymptotic behavior. We call them semi-compact structures, and they have an important difference, when compared to the compact structures discussed in Ref. [21]. The key point here is that the semi-compact solutions that we introduce have asymptotic behavior similar to the standard spin textures, but they behave differently as one approaches the core of the structure. We understand that this is important behavior, because it help us show how to control the inner core of skyrmions.

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We organize the work as follows: in the next section we follow Refs. [23,24] and discuss three distinct models describing a single real scalar field in two spatial dimensions, as suggested in Ref. [25]. We describe explicit solutions, and study their stability. In Section 3, we deal with the topological charge density and study the topology and the skyrmion number. We end the work in Section 4, where we include our comments and conclusions.

2. Analytical procedure

We start focusing on the localized spin textures that appear in magnetic materials. We suppose that the material is homogeneous along the $\hat{z}$ direction, with the magnetization $\mathbf{M}$ being in general a three-component vector with unit modulus that depends on the planar coordinates, such that $\mathbf{M} = \mathbf{M}(x, y)$ and $\mathbf{M}\cdot\mathbf{M} = 1$.

To describe skyrmions, we introduce the skyrmion number, which is a conserved topological quantity, given by

$$Q = \frac{1}{4\pi}\int_{-\infty}^{\infty} dxdy \quad \mathbf{M}\cdot\partial_x\mathbf{M}\times\partial_y\mathbf{M}. \tag{1}$$

In this work we concentrate on helical excitations, with the magnetization $\mathbf{M} = \mathbf{M}(r)$ only depending on the radial coordinate, being now a two-dimensional vector, orthogonal to the radial direction. In cylindrical coordinates, the helical excitations impose that $\mathbf{M}\cdot\hat{r} = 0$, so one allows that the magnetization has the form

$$\mathbf{M}(r) = \hat{\theta}\cos\Theta(r) + \hat{z}\sin\Theta(r), \tag{2}$$

With this decomposition, we can then describe $\Theta(r)$ as the only degree of freedom to be used to model the magnetic excitation in the magnetic material. If we now use Eq. (1) with the (r, θ) variables, the above magnetization can be used to obtain the skyrmion number. It leads to

$$Q = \frac{1}{2}\sin\Theta(0) - \frac{1}{2}\sin\Theta(\infty). \tag{3}$$

This result shows that the topological profile of the solution $\Theta(r)$ is related to its value at the origin, and the asymptotic behavior for $r\to\infty$.

In order to model skyrmion-like solutions analytically, we take advantage of the recent study [23,24] and consider

$$\Theta(r) = \frac{\pi}{2}\phi(r) + \delta, \tag{4}$$

where δ is a constant phase, which we use to control the value of the magnetization at the center of the magnetic structure. Also, we suppose that the scalar field ϕ is homogeneous and dimensionless quantity which is described by the planar system investigated in Ref. [25]. In this case, the Lagrange density $\mathcal{L}$ has the form

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\nabla\phi\cdot\nabla\phi - U(\phi), \tag{5}$$

where dot stands for time derivative, and ∇ represents the gradient in the (x, y) or (r, θ) plane. We search for time independent and spherically symmetric configuration, $\phi = \phi(r)$, and consider $U = U(r;\phi)$ in the form

$$U(r,\phi) = \frac{1}{2r^2}P(\phi), \tag{6}$$

with $P(\phi)$ an even polynomial which contains non-gradient terms in ϕ. In this case, the field equation becomes

$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} - \frac{1}{2}\frac{dP}{d\phi} = 0, \tag{7}$$

and the energy for static solution $\phi(r)$ is given by

$$E = 2\pi\int_0^{\infty} rdr\rho(r), \tag{8}$$

with $\rho(r)$ being the energy density, such that

$$\rho(r) = \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{2r^2}P(\phi). \tag{9}$$

The point here is that we can describe explicit models, using distinct functions for the Polynomial $P(\phi)$. We do this for three distinct models in the next subsections. Before doing that, however, we note that for $\phi\to\phi_v$, with ϕ_v being constant and uniform, the energy density vanishes if ϕ_v is a minimum of $P(\phi)$ such that $P(\phi_v) = 0$ [23]. In this case, ϕ_v is a ground state and will guide us toward the construction of models.

Another issue is that the model one uses engender scale invariance; see, for instance, the equation of motion (7). An interesting route to describe magnetic skyrmions is to break scale invariance, and this can be achieved via the introduction of Dzyaloshinskii-Moriya interactions (see, e.g., [19] and references therein); another route is to describe skyrmions on a lattice (see, e.g., [28] and references therein). In this work we will keep using the model (5) to describe the magnetization since it will lead to analytical solutions. In this case, although we cannot use results to describe quantitative properties, we can still use them qualitatively and, more importantly, we can explore the topological properties of the solutions, because the topology follows from their asymptotic behavior, and may appear despite the breaking of the scale invariance.

2.1. Non-topological solutions

The first model that we describe is governed by the polynomial $P(\phi)$, given by

$$P(\phi) = n^2\phi^2(\phi^{-2/n} - 1). \tag{10}$$

Here $n = 3,\ 5,\ 7,\ldots$ is an odd integer. This model follows the suggestion of Ref. [26], concerning the presence of one-dimensional, compact lump-like structures there studied. We depict this polynomial $P(\phi)$ in Fig. Fig. 1 for three distinct values of n, to show how it behaves as one changes such parameter. We see that the polynomial has a local minimum at $\phi_0 = 0$, and two zeroes at $\phi_\pm = \pm 1$, irrespective of the value of n. We can write the equation of motion as

$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} + n^2\phi - n(n-1)\phi^{1-2/n} = 0. \tag{11}$$

One then searches for solutions related to the ground state $\phi_0 = 0$. The investigation has led us with the interesting family of analytical solutions

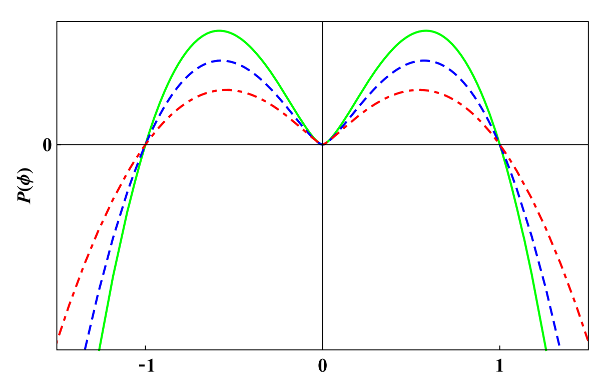

Fig. 1. The polynomial (10), depicted for $n = 3,\ 5$, and 7, with dot-dashed (red), dashed (blue) and solid (green) lines, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)