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Topology optimization of Halbach magnet arrays using isoparametric projection

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ABSTRACT

Topology optimization using isoparametric projection for the design of permanent magnet patterns in Halbach arrays is proposed. Based on isoparametric shape functions used in the finite element analysis, the permanent magnet strength and magnetization directions in a Halbach array are simultaneously optimized for a given design goal. To achieve fabrication feasibility of a designed Halbach magnet array, two design schemes are combined with the isoparametric projection method. First, a penalization scheme is proposed for designing the permanent magnets to have discrete magnetization direction angles. Second, an extrusion scheme is proposed for the shape regularization of the permanent magnet segments. As a result, the method systematically finds the optimal permanent magnet patterns of a Halbach array considering manufacturing feasibility. In two numerical examples, a circular shaped permanent magnet Halbach array is designed to minimize the magnitude of the magnetic flux density and to maximize the upward direction magnetic flux density inside the magnet array. Logical extension of the method to the design of permanent magnet arrays in linear actuators is provided, where the design goal is to maximize the actuator magnetic force.

1. Introduction

A Halbach array is a special pattern or arrangement of permanent magnets (PMs) having different magnetization directions. The one-sided magnetic flux effect was first discovered by Mallinson [1] and deeply studied by Halbach [2,3]. Since then, Halbach array PMs have been extensively investigated for a wide range of applications with the hope of improved system performance. As an example, linear actuators with cylindrical Halbach arrays were proposed, and the performance of such electromechanical devices was analyzed in [4]. Halbach arrays with trapezoidal shaped PMs were proposed for an actuator to increase the magnetic flux density [5]. Halbach arrays have been also applied in the design of electric motors [6,7]. In [6], cylindrical Halbach arrays were employed for brushless servo motors, and it was shown that the use of a Halbach array can increase the motor torque up to 33%. In [7], a dipole magnet array was utilized in the development of a novel electric machines. Vibration energy harvesters have also been built using Halbach arrays. An energy harvester with Halbach array was first proposed in [8], and the device output power was improved using shaped PMs and double arrays [9]. Beyond actuator, motor, and energy harverster applications, Halbach arrays have also been applied in biomedical application [10–12] and magnetic refrigeration [13,14]. In [10,11], Halbach arrays were investigated and designed for use of magnetic targeting of drugs to diseased tissues. A cell separator using Halbach arrays were proposed and studied in [12]. In [13], Halbach cylinder arrays were applied for magnetic refrigeration based on magneto-caloric effect. Rotary magnetic refrigerators were designed using multi-polar concentric Halbach arrays in [14].

The use of a Halbach array does not always guarantee performance improvement in devices. To achieve improvement, the pattern of PM magnetization directions and the shape of the PMs in the array should be carefully designed. The optimal dimensions of a Halbach array was studied in 16 segmented cylinders [15] and continuous ideal remanent flux density [16] cases. The design process to find optimal magnet and iron segments was proposed for the application of magnetic refrigeration in [17]. Here, the segmentation of the optimized magnet and iron was done manually, and the magnetization direction of the individual segments was automatically found using an optimization algorithm. Recently, a versatile design approach for finding the optimal shape of uniformly magnetized segments in two dimensions was proposed

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based on the reciprocity theorem [18,19]. Here, the optimal segment borders and splitting angles are automatically determined through analytical treatments. This approach is expected to satisfy global optimality conditions if the objective functional is linear. However, an analytic-based approach might not be appropriate when optimization objectives and constraints are severely complex.

In this paper, we propose a design methodology for PMs in Halbach arrays using a topology optimization technique. Note that topology optimization can be effectively applied for the PM design even in complex optimization problems (e.g. PM design for magnetorheological (MR) fluid cooling [20]) due to its compatibility with finite element analysis. The use of topology optimization methods can assist in effectively determining a PM magnetization pattern and the shape of individual PM segments without requiring designer's intuition. Based on the numerical analysis procedure, topology optimization finds the optimum solution, which may result in a better design than simple parametric optimization since topology optimization investigates a wider design space without assuming a fixed initial design. Fig. 1 shows an abstract schematic representation of the optimization problem settings. Here, the pattern of the PM magnetization directions is determined for a given design goal such as maximizing the magnetic flux density at a specific position.

Topology optimization of Halbach arrays has been performed in [21,22]. In [21], topology optimization was first applied in the design of Halbach arrays. However, the methodology proposed in [21] is limited in that the design result only provides for continuous PM segment magnetization directions instead of discrete directions, which is limited in terms of magnet fabrication. To overcome this limitation and design PMs with discrete magnetization direction angles, the interpolation scheme between the PM residual flux density and design variables was modified [22]. Nonetheless, the design methodology proposed in [22] might not be systematic enough for various design requirements of Halbach arrays since the set of discrete PM magnetization directions is limited to four (in Cartesian coordinates), and additional design variables are needed for more directions. Furthermore, some design variables used in [22] are unnecessary in specific design cases (e.g., d_y is ignored when $\rho_x = 1$ and $\rho_y = 1$); refer to [22] for details.

Accordingly, this paper applies topology optimization with isoparametric projection, as explained in [23], to design Halbach magnet arrays. In [23], a new topology optimization method was proposed for the simultaneous design of magnet topology (i.e., density) and orientation using an isoparametric projection technique. By exploiting isoparametric shape functions that are generally used in the finite element method, the design variables in local coordinates are projected into the orientation vector in global coordinates. This approach was successfully applied to traditional beam/truss design problems in [23], where the optimal structural topology and anisotropic composite material fiber reinforcement orientation was found. In this work, the isoparametric projection method is extended to the simultaneous design of PM shape and magnetization direction of magnetic segments in a Halbach array.

To achieve fabrication feasibility of the PM design result, two design schemes that can be combined with the isoparametric projection method are proposed. The first scheme is the penalization of intermediate PM magnetization direction angles. In the design of a Halbach array PM, discrete PM magnetization direction angle sets of are preferred for ease of fabrication. Thus, intermediate magnetization angles are penalized by modifying the isoparametric element shape in the global coordinates. Simple modification of the selected element shape enables us to obtain design results with various desired discrete PM angle sets such as four, six, or eight directions in rectangular or cylindrical coordinates. In this work, the discrete PM angle set is predetermined as regularly distributed. The proposed approach using the isoparametric projection allows for an irregularly distributed angle set, and thus it is also possible to determine the angle set automatically through the optimization algorithm.

A second extrusion scheme is also proposed for PM shape regularization, which again might be preferred to facilitate easy fabrication of PM segments. Specifically, in the design of a rectangular Halbach array, design variables are located only at the bottom linear boundary of the rectangular array outline. The associated design results are then extruded in the vertical direction to fill the corresponding rectangular design domain, which naturally assigns PM segments a rectangular shape along the magnetization direction. In the case of a cylindrical Halbach array, the design domain is the outer circumferential boundary of the annular array and the design results are extruded to the innermost circumferential boundary of the annulus along the radial direction. This approach enables us to design PM pieces with tangential and radial directions and which have manufacturing advantages. It should be noted that the proposed extrusion scheme is applicable for the design of thin and long shaped PMs that are generally employed for motor and actuator applications. If the extrusion length becomes too long, the performance loss by dimension reduction will exceed the manufacturing advantages. For this case, full two dimensional topology optimization [20] only with the penalization scheme might be more appropriate.

The proposed isoparametric projection method with penalization and extrusion schemes are applied to two case studies including the design of cylindrical Halbach magnet arrays and PM arrays in linear actuators. In the first case of a cylindrical Halbach array, two design examples are provided. The design goal in the first example is to obtain a magnet arrangement that minimizes the magnitude of magnetic flux density inside the Halbach array. In contrast, the aim of the second problem is to maximize the upward direction magnetic flux density inside the Halbach array. For the

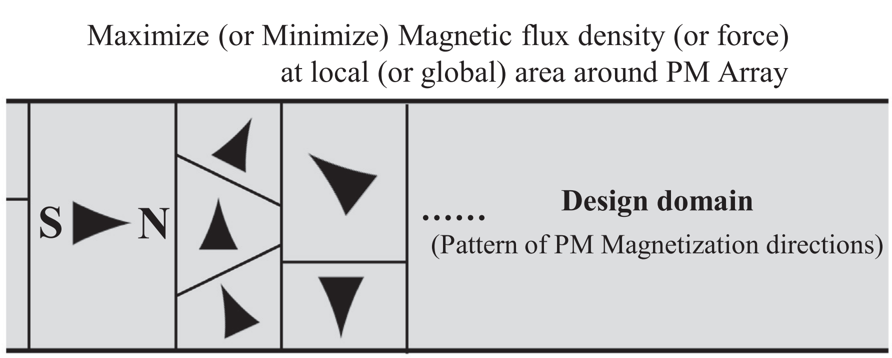

Fig. 1. Abstract schematic representation of a Halbach array design optimization problem.