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Reply to "Comment on 'Performance of Halbach magnet with finite coercivity'."

In the comment [1] on our work [2], the author argues that *B-H* relations are not suitable for the analysis of demagnetization effects because " *B-H* functions are sensitive to the shape of magnetic element". In the context of macroscopic electromagnetism the concept of local constitutive relation does not involve any shape effect. A constitutive *B-H* relation expresses the link between the fundamental field $\boldsymbol{B}$ and the macroscopic field $\boldsymbol{H}$ in a particular point [3]. In this context the entity of point signifies a macroscopically infinitesimal region which is still large enough for the microscopic effects to be averaged out. The issues raised by the author of [1] seems to arise from a confusion with the concept of demagnetization field and demagnetization factors. These concepts are relevant to the mathematical procedures used to correctly interpret the relation between experimental magnetization data and applied field considering the shape of the sample. To be clear, we emphasize that in ref. [1] and in the present reply we use the term "demagnetization" to signify a situation in which the remanence vector of a region of the permanent magnet cannot retain its original direction or norm.

Moreover the author of [1] expresses concerns over the singular behavior at $H = H_c$ of the model *B-H* curves used in [2]. These *B-H* curves, which are an idealization of the actual magnetic behavior of permanent magnets, do not actually present any issue with the computational procedure adopted in [2].

It is worthwhile to briefly review here the procedure employed by Xu et al. in [4] and the procedure employed by us. In the following, bold symbols represent vector quantities, while the non-bold symbols represent scalar quantities. The symbols $\parallel$ and $\perp$, denotes the two directions parallel and normal to the easy magnetization axis. The unit vector in the direction parallel to the easy axis, denoted by $\hat{\boldsymbol{e}}_{\parallel}$, is given by:

$$\hat{\boldsymbol{e}}_{\parallel} = \cos, (\phi)\hat{\boldsymbol{e}}_r + \sin(\phi)\hat{\boldsymbol{e}}_{\phi} \tag{1}$$

Here r and ϕ denote the radial and angular coordinates, respectively, and $\hat{\boldsymbol{e}}_r$ and $\hat{\boldsymbol{e}}_{\phi}$ denote the corresponding unit vectors. The unit vector of the *x-y* plane pointing in the direction normal to the easy axis, denoted by $\hat{\boldsymbol{e}}_{\perp}$, is given by:

$$\hat{\boldsymbol{e}}_{\perp} = \sin(\phi)\hat{\boldsymbol{e}}_r - \cos(\phi)\hat{\boldsymbol{e}}_{\phi} \tag{2}$$

Procedure used by Xu et al

The procedure starts with the original magnetization distribution $\boldsymbol{M}_0 = M_R\hat{\boldsymbol{e}}_{\parallel}$, corresponding to a given remanence. Here $M_R = (1/\mu_0)B_{\text{rem}}$ denotes the norm of the original magnetization. The magnetization volumetric current $\boldsymbol{J}$ is calculated from $\boldsymbol{M}$ using:

$$\boldsymbol{J} = \nabla \times \boldsymbol{M} \tag{3}$$

The magnetization surface current $\boldsymbol{K}$ at the interface between two regions is given by:

$$\boldsymbol{K} = \boldsymbol{M} \times \hat{\boldsymbol{n}} \tag{4}$$

Here $\hat{\boldsymbol{n}}$ denotes the unit vector normal to the discontinuity surface. The magnetic flux density distribution $\boldsymbol{B}$ can be calculated from the current densities using the Biot-Savart law, i.e. performing a numerical integration. The magnetization distribution is then modified using $\boldsymbol{B}$ according to the $\boldsymbol{M}(\boldsymbol{B})$ constitutive relation, expressed in terms of two experimentally determined functions $M_{\parallel}(B_{\parallel})$ and $M_{\perp}(B_{\perp})$. A normalization is performed when this relation would lead to a magnetization with a norm higher than the saturation magnetization M_S. Here we denote by $\check{\boldsymbol{M}}$ the non-normalized magnetization vector, expressed by:

$$\check{\boldsymbol{M}} = \hat{\boldsymbol{e}}_{\parallel}M_{\parallel}(B_{\parallel}) + \hat{\boldsymbol{e}}_{\perp}M_{\perp}(B_{\perp}) \tag{5}$$

The constitutive relation used by Xu. et al. is then synthetically expressed by:

$$\begin{cases} \boldsymbol{M}(\boldsymbol{B}) = \check{\boldsymbol{M}} & \text{for} \quad \parallel \check{\boldsymbol{M}} \parallel \le M_s \\ \boldsymbol{M}(\boldsymbol{B}) = \check{\boldsymbol{M}}(M_s/\parallel \check{\boldsymbol{M}} \parallel) & \text{for} \quad \parallel \check{\boldsymbol{M}} \parallel > M_s \end{cases} \tag{6}$$

The Biot-Savart law is then employed again to compute $\boldsymbol{B}$ from the modified $\boldsymbol{M}$ distribution. The procedure is repeated iteratively until a self-consistent magnetization distribution is obtained.

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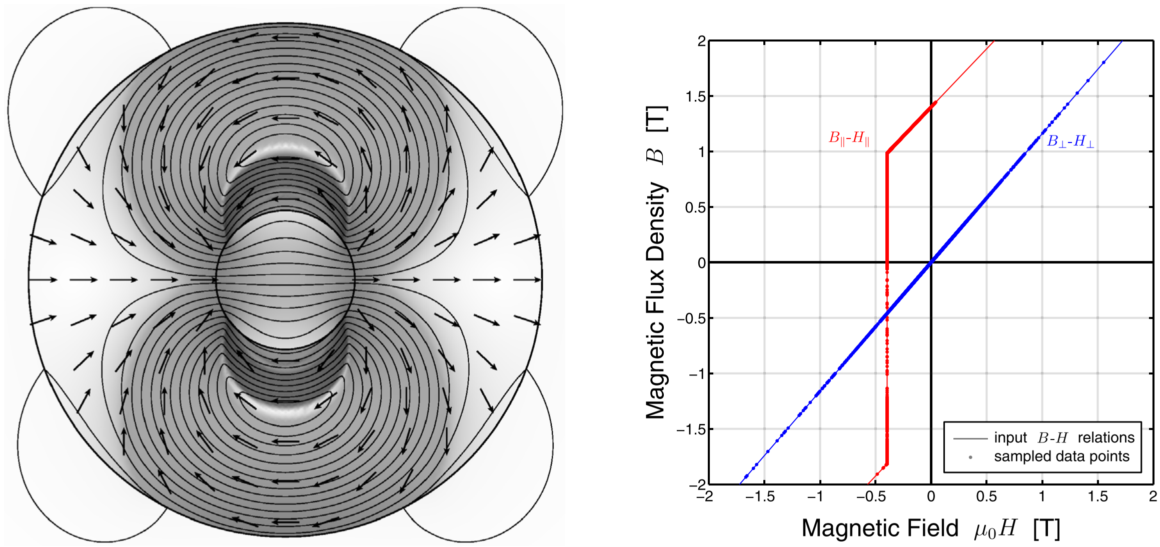

(a) Halbach cylinder (b) B-H relation.

Fig. 1. 1a: Halbach cylinder affected by non-linear demagnetization. The norm of the flux density is indicated by the background color, darker shades corresponding to a higher norm. The field lines of $\boldsymbol{B}$ are shown as solid lines. In the Coulomb gauge these are equivalent to level curves of A_z. 1b: constitutive relation used to perform this simulation. The horizontal axis corresponds to $\mu_0 H$, and is therefore expressed in teslas. The components parallel and normal to the easy axis are shown as red and blue solid lines, respectively. The dots indicate the values of field and flux density obtained from the computation by sampling 5000 random points of the permanent magnet region. The figure shows a perfect match between the computed values and the input B-H curves for the parallel and perpendicular directions. This computation corresponds to the parameters: B_r=1.4 T, $\mu_r^{\|} = 1.05$, $\mu_r^{\perp} = 1.16$ and $\mu_0 H_c = 0.4$ T.

Procedure used by Insinga et al

The starting point is the constitutive relation $\boldsymbol{B} = \boldsymbol{f}(\boldsymbol{H})$, for simplicity assumed to be piecewise linear:

$$\begin{cases} B_{\|} = \mu_0 \mu_r^{\|} H_{\|} + B_{\text{rem}} & \text{for} \quad H_{\|} > -H_c \quad \text{(linear region)} \\ B_{\|} = \mu_0 \mu_r^{\|} H_{\|} - B_{\text{rem}} & \text{for} \quad H_{\|} < -H_c \quad \text{(reversed linear region)} \\ B_{\perp} = \mu_0 \mu_r^{\perp} H_{\perp} \end{cases} \tag{7}$$

This relation is a good first approximation to the behavior of actual hard magnet materials. The parallel and normal components of this constitutive relations are represented in Fig. 1b by the red and blue solid lines, respectively. Even though the constitutive relation expressed by Eq. (7) admits multiple values of $B_{\|}$ for $H_{\|} = -H_c$, the inverse relation $\boldsymbol{f}^{-1}$ is uniquely defined. Since our procedure, described below, uses the inverse relation $\boldsymbol{H} = \boldsymbol{f}^{-1}(\boldsymbol{B})$, this multiplicity does not present any difficulty.

We adopt the magnetic vector potential formulation. For a two dimensional system the only non-zero component of the magnetic vector potential, $\boldsymbol{A}$, is the z component. Since the magnetic flux density, $\boldsymbol{B}$, is defined as $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, it can only be a solenoidal field, thus automatically obeying Gauss's law for magnetism: $\nabla \cdot \boldsymbol{B} = 0$. Using the inverse constitutive relation, $\boldsymbol{H} = \boldsymbol{f}^{-1}(\boldsymbol{B})$, Ampere's law can be written as:

$$\nabla \times (\boldsymbol{f}^{-1}(\nabla \times \boldsymbol{A})) = \boldsymbol{J}_{\text{free}} \tag{8}$$

where $\boldsymbol{J}_{\text{free}}$ denotes the free electrical current density, i.e. not associated with the magnetization currents. For the system under analysis $\boldsymbol{J}_{\text{free}}$ is clearly zero. Eq. (8) can be solved with standard numerical techniques. We use the commercial FEM simulation software COMSOL Multiphysics. The result of one example simulation is shown in Fig. 1a. The norm of the flux density is indicated by the background color, darker shades corresponding to a higher norm. The field lines of $\boldsymbol{B}$ are shown as solid lines. The B-H curves shown as solid lines in Fig. 1b corresponds to the parameters used for this simulation. The dots indicated the values of the parallel and normal component of $\boldsymbol{H}$ and $\boldsymbol{B}$ obtained from the simulation by sampling 5000 random points inside the permanent magnet region. As can be seen, the results are perfectly consistent with the input constitutive relation.

Short discussion

Evidently, our approach automatically guarantees that the distributions of $\boldsymbol{B}$ and $\boldsymbol{M}$ are consistent with the constitutive relation. Moreover, it seems necessary to point out that the Biot-Savart law is derived from the same starting point, i.e. Gauss's law for magnetism and Ampere's law. Using the definition:

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \tag{9}$$