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MR fingerprinting reconstruction with Kalman filter

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ABSTRACT

Magnetic resonance fingerprinting (MR fingerprinting or MRF) is a newly introduced quantitative magnetic resonance imaging technique, which enables simultaneous multi-parameter mapping in a single acquisition with improved time efficiency. The current MRF reconstruction method is based on dictionary matching, which may be limited by the discrete and finite nature of the dictionary and the computational cost associated with dictionary construction, storage and matching.

In this paper, we describe a reconstruction method based on Kalman filter for MRF, which avoids the use of dictionary to obtain continuous MR parameter measurements. With this Kalman filter framework, the Bloch equation of inversion-recovery balanced steady state free-precession (IR-bSSFP) MRF sequence was derived to predict signal evolution, and acquired signal was entered to update the prediction. The algorithm can gradually estimate the accurate MR parameters during the recursive calculation. Single pixel and numeric brain phantom simulation were implemented with Kalman filter and the results were compared with those from dictionary matching reconstruction algorithm to demonstrate the feasibility and assess the performance of Kalman filter algorithm.

The results demonstrated that Kalman filter algorithm is applicable for MRF reconstruction, eliminating the need for a pre-define dictionary and obtaining continuous MR parameter in contrast to the dictionary matching algorithm.

1. Introduction

MR fingerprinting, or MRF, offers efficient means to obtain multiple quantitative maps, such as T1, T2 and off-resonance, within a single sequence by matching the acquired signal with a pre-defined dictionary [1]. In MRF, a pseudorandom TR and flip angle is used to generate unique signal evolution for each tissue, which reflects the inherent characters of the tissue. The acquired signal evolution is looked up in a dictionary generated by Bloch simulation with the TR and flip angle patterns same as that used in acquisition. It provides a promising approach for fast acquisition and quantitative imaging. Currently MRF has been explored in several applications including quantitative abdominal imaging [2], and cardiac imaging [3].

In the current dictionary-matching algorithm, the dictionary needs to be regenerated once the sequence or acquisition parameters change, which causes extra calculation time and memory cost. On another hand, dictionary matching introduces round off error because of discrete parameter space. Tradeoff between dictionary size and quantization error needs to be considered. Several studies have investigated the acquisition sequence, dictionary generation, and matching algorithm to overcome these problems. Fast imaging with steady state precession (FISP) sequence was used instead of balanced steady state free-precession (bSSFP) sequence to reduce dictionary size by eliminating off-resonance term [4]. Tree-Structured Vector Quantizer (TSVQ) reduced dictionary size by applying *k*-means clustering to training set recursively instead of taking grid points in the parameter space [5]. Maximum likelihood (ML) reconstruction was explored by taking conventional dictionary results as initializations and approaching ML optimum in the subsequent iterations to improve accuracy [6]. However, all these reconstruction methods were based on the framework of dictionary matching, so they more or less encounter the inherent problems such as the tradeoff between the dictionary size and the accuracy. If more parameters maps are considered in the signal model, the dictionary size will increase exponentially, which further exacerbates the problem. Moreover, the size of dictionary is strictly limited in some applications such as cardiac triggered scan, which need a new dictionary every time because the TR patterns are dictated by heart rate [3].

In this paper, we introduce a Kalman filter based reconstruction algorithm to recursively derive the MR parameters from acquired MRF data with the signal model of acquisition sequence, providing an

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alternative to dictionary matching to avoid problems associated with dictionary generation. A preliminary result was presented in [7].

2. Theory

2.1. Kalman filter algorithm

The Kalman filter is an algorithm using a series of measurements to estimate the state of the system over time, resulting in a more accurate estimation than a single measurement [8]. The Kalman filter was first implemented in the Apollo program, where it played an essential role in the Apollo navigation system [9]. It was then widely used in many fields, including the Global Positioning System (GPS) [10] and other fields of signal processing. In most applications, the internal state has more degrees of freedom than the measured parameters. However, based on the measurements and the underlying physics governing the system, the Kalman filter combines all the information and provides an accurate estimation of the unknown parameters of the system. Taking the space shuttle as an example, the acceleration and angles between stars can be obtained by an onboard inertial navigator [9]. When using the additional information from the laws of motion, the Kalman filter can obtain the position and velocity of the space shuttle, making it stay in the right orbit.

It is a very similar situation when it comes to MRF, where the physics laws governing the signal model is no longer laws of motion, but Bloch equation. If we put unknown parameters (spin–lattice relaxation time $T1$, spin–spin relaxation time $T2$, off-resonance frequency df) and Mz, as well as the measured MR signal (Mx, My) together to form a joint state vector [11], it is possible for Kalman filter to simultaneously track the evolution of magnetization vectors and estimate the parameters underlying the physics model. Thus, we first define the unknown parameter vector $p_k = [T1, T2, df]^T$, we then have the joint state vector

$$S_k \triangleq \left[M_k^T, p_k^T\right]^T = [Mx_k, My_k, Mz_k, T1, T2, df]^T \tag{1}$$

where Mx_k, My_k, Mz_k represent magnetization along three axis at time point k.

To use Kalman filter algorithm, two functions are needed. One is the system dynamic function $f_k(S)$ which describe the relationship between the joint state vector at an arbitrary time point and that at subsequent time point. The other is the observation function H which describes how the observation $y_k = [Mx_k, My_k]^T$ is linked to the joint state vector S_k. We have

$$\begin{aligned} S_k &= f_k(S_{k-1}) + v_k \\ y_k &= HS_k + w_k \end{aligned} \tag{2}$$

where vector $v_k \in R_{6\times1}$ and $w_k \in R_{2\times1}$ represent the additive Gaussian white process noise and measurement noise, respectively [12]. The process noise v_k and the measurement noise w_k are zero mean multivariate normal distributions with covariance matrices Q_k and R_k, respectively. Generally speaking, the process noise v_k represents other effects that are not modeled (to reduce the complexity of model), and the measurement noise w_k represents the noise associated with the measurement. These noise levels are estimated empirically before the recursive algorithm and have a significant effect on the calculation of Kalman gain, as described in Appendix A. More specifically, the process noise v_k represents any other effects than T1, T2 relaxation and the precession due to off resonance, which are not included in our signal model, such as RF imperfection. In the simulation here, since the standard deviation of measurement noise is preset, the measurement noise matrix R_k is also known, but in practice it needs to be empirically estimated.

The Kalman filter is a recursive algorithm, including two steps: predict and update. As shown in Fig. 1, in the predict step, the system dynamic function $f_k(S)$ is used to predict the joint-state vector S_k at the next time frame $\mathcal{N}(\hat{S}_{k|k-1}, P_{k|k-1})$ (prediction, also known as priori). In the update step, an observation y_k' at k step is used to adjust the prediction of the predict step and estimate the joint state vector $\mathcal{N}(\hat{S}_{k|k}, P_{k|k})$ (estimation, also known as posteriori). Please note that the prediction and estimation are represented by mean and covariance and are deterministic in the recursive calculation, while the joint-state vector S_k in Eq. (2) is an unknown random variable. Similarly, the actual observation y_k' is the collected data and deterministic while the observation y_k in Eq. (2) is also an unknown random variable. The Kalman gain K is a weight that determines how to combine the observation and the prediction, which is calculated from the covariance matrices of the observation and the prediction. The detailed Kalman filter algorithm including how to calculate Kalman gain is described in Appendix A for further reading. When the predicting-updating cycle is repeated, the covariance of joint-state vector gradually decreases, and the estimated value of joint-state vector converges to its true value. In the next sections, we will describe how the system dynamic function and the observation function are derived.

2.2. System dynamic function

The system dynamic function describes how the magnetization vector and the MR parameters change as a function of time, which is based on Bloch equation. It contains the physics model underlying the MRF signal, and it is used in the predict step of Kalman filter cycle. As such, it represents the basic physics model in the Kalman filter algorithm framework.

To derive the system dynamic function, we can focus on the signal at an arbitrary time point, $k-1$, and the one at the subsequent time point, k. From the middle of the TR($k-1$), the magnetization vector evolves

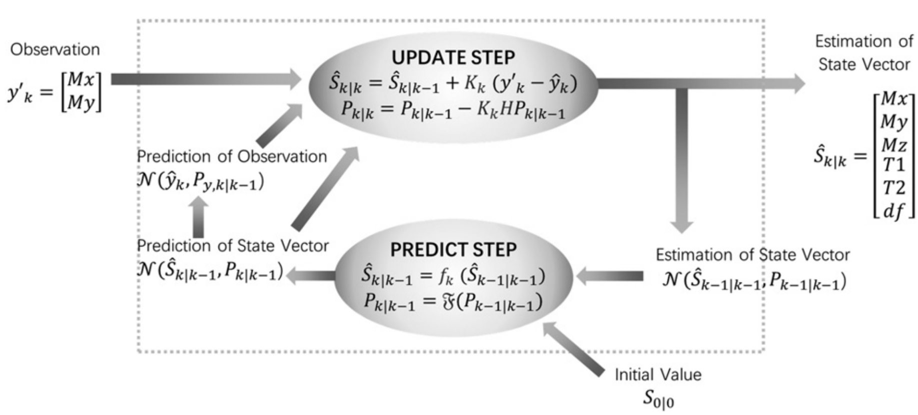

Fig. 1. Flow chart of Kalman filter algorithm.