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Influence of electron-phonon interaction on quantum phase transition in a triangular triple quantum dot

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ABSTRACT

We investigate the quantum phase transition in triple quantum dot system with triangular geometry, in which one of the dots is connected to metallic leads and electrons in the dot interact with a local phonon mode. The influence of electron-phonon interaction on the quantum phase transition between local moment phase and Kondo screened strong coupling phase at the particle-hole symmetric point is studied based on the analytical arguments and the numerical renormalization group method. The results show that the critical value of tunnel-coupling between side dots decreases with the increase of electron-phonon coupling in "spin Kondo" regime. Furthermore, at a certain critical value of electron-phonon coupling, there appears only strong coupling phase, irrespective of tunnel-coupling between dots. The study of the influence of electron-phonon interaction on the quantum phase transition in triple quantum dot has the great importance for clarifying the mechanism of Kondo screening in the system.

1. Introduction

Triple quantum dot (TQD) with triangular structure plays an important role on various fascinating phenomena in the condensed matter physics. For example, when electrons move around a three-dot loop in magnetic field, one can observe quantum coherence phenomena such as Aharonov-Bohm effect [1,2]. TQD with identical couplings also causes frustration [3].

Single triangle is also a fundamental unit of the triangular and Kagome lattices. In these systems, the geometrical frustration affects significantly the magnetic properties and the behavior at the Mott–Hubbard metal–insulator transition [4,5]. Another interesting example is the triangular trimer of Cr atoms placed upon an Au surface [6–8], expected to show a non-Fermi-liquid behavior due to the multi-channel Kondo effect [9,10].

Recently, the quantum phase transition between Kondo screened strong coupling (SC) phase and local moment (LM) phase in triple quantum dot system has attracted much interest [11]. In TQD with weak inter-dot couplings, such quantum phase transition is the consequence of competition between direct and indirect exchange interactions between two side dots [12–14]. The latter is the indirect RKKY-type interaction mediated by the Kondo singlet formed between embedded dot and leads. Therefore, the sign of the actual effective exchange interaction energy between two side dots depends on which of the two interactions prevails. The quantum phases of the system depend upon the sign of the effective exchange interaction energy between two side dots.

An important problem in interacting many-particle systems is to understand the interplay between electron-electron and electron-phonon interactions [15,16]. Especially, in the semi-conductor quantum dots, the effect of electron-phonon interaction is significant for Kondo physics due to electron-electron interaction. In general, due to the attractive interaction mediated by phonon, the effective electron-electron interaction may be repulsive or attractive depending upon the strength of electron-phonon coupling. For the repulsive interaction with weak electron-phonon coupling, there occurs the conventional Kondo effect (so called "spin Kondo effect") in which the local spin of a quantum dot is screened by spin density in conduction band. For the attractive interaction with strong electron-phonon coupling, the isospin of a quantum dot is, however, screened by isospin density in conduction band, which is called "charge Kondo effect" [17]. On the theoretical side, the role of electron-phonon interactions has been studied in the framework of the Holstein and Anderson-Holstein models [18–24].

In TQD with electrons in the embedded dot (connected to metallic leads) interacting with a local phonon mode, Kondo physics and quantum phase transition show a new behavior depending upon the strength of electron-phonon coupling. For weak electron-phonon interaction, since there appears the conventional Kondo effect, side dots become RKKY-coupled by the embedded dot. However, for strong electron-phonon interaction, which makes the effective electron-elec-

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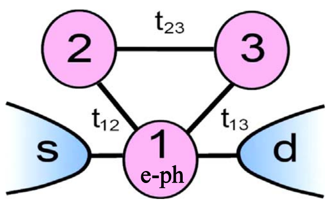

Fig. 1. Geometrical structure of the triple quantum dot system. The embedded dot (dot 1) is connected to source (s) and drain (d) leads and the electrons in it interact with a local phonon mode.

tron interaction attractive, side dots don't have RKKY coupling due to disappearance of the conventional Kondo effect.

In this work, we study the effect of electron-phonon interaction on the quantum phase transition between LM phase and Kondo screened SC phase in triple quantum dot system with triangular geometry, in which one of the dots is connected to metallic leads. In our system, each dot has the particle-hole symmetry and electrons in the embedded dot interact with a local phonon mode. Our discussion is restricted to the "spin Kondo" regime for weak electron-phonon coupling. In particular, we show how the phase diagram, representing the quantum phase transition between Kondo screened SC phase and LM phase, is modified as electron-phonon coupling increases.

2. Model and analytical argument

We consider the triangular triple quantum dot system with one of dots linearly coupled to a local phonon mode, as shown in Fig. 1. The corresponding Hamiltonian for the model is then given by

$$H = H_{TQD} + H_E + H_T + H_{ph} + H_{e-ph}, \tag{1}$$

$$H_{TQD} = \sum_{i=1,2,3} [\varepsilon_i (n_{i\uparrow} + n_{i\downarrow}) + U_i n_{i\uparrow} n_{i\downarrow}] + \sum_{i<j,\sigma} (t_{ij} d_{i\sigma}^{+} d_{j\sigma} + h.\ c.), \tag{2}$$

$$H_E = \sum_{\alpha=s,d} \sum_{\sigma} \sum_{k} \varepsilon_k C_{\alpha k\sigma}^{+} C_{\alpha k\sigma}, \tag{3}$$

$$H_T = \sum_{\alpha,k,\sigma} [V_{\alpha k} d_{1\sigma}^{+} C_{\alpha k\sigma} + \text{H.c.}], \tag{4}$$

$$H_{ph} = \omega_0 (b^+ b + 1/2), \tag{5}$$

$$H_{e-ph} = g(b^+ + b)(n_{1\uparrow} + n_{1\downarrow} - 1), \tag{6}$$

where H_{TQD} describes the isolated three quantum dots with single energy ε_i, coulomb interaction U_i, and inter-dot couplings t_{ij} and H_E the leads, essentially being non-interacting but macroscopic metal. Here, $d_{i\sigma}^{+}$ and $C_{\alpha k\sigma}^{+}$ are the creation operators for electrons in quantum dots and leads, respectively, and $n_{i\sigma} = d_{i\sigma}^{+} d_{i\sigma}$. The hybridization term H_T represents tunnel-coupling between dot 1 and leads with the strength $V_{\alpha k}$. In the case that dot 1 is coupled to a local phonon mode, the added terms, H_{ph} and H_{e-ph} describes free phonon and electron-phonon interaction, where b^+ creates a phonon of frequency ω_0 and g is the electron-phonon coupling.

For the interpretation of the model, we first consider the case that Kondo singlet state is formed between embedded dot and leads. In this case the density of states on the embedded dot has the Kondo resonant peak with T_k width and $1/T_k$ height around Fermi level, leading to the formation of "conduction electron cloud" in the dot 1. Then, side dots 2 and 3 become RKKY-coupled by "conduction electron cloud". Since the RKKY interaction is responsible for ferromagnetic coupling between side dots, this competes with direct exchange interaction responsible to antiferromagnetic coupling. In this paper, we neglect much narrower gap inside Kondo peak, because its contribution to the RKKY interaction is significantly small when tunnel-couplings between embedded dot and side dots are weak [25].

The Hamiltonian describing the RKKY interaction between side dots mediated by "conduction electron cloud" can be written as follows [26]:

$$H_{RKKY} = J_{RKKY} \vec{S_2} \cdot \vec{S_3}, \tag{7}$$

$$J_{RKKY} = J_2 J_3 \chi(\vec{r}, \vec{r}'), \tag{8}$$

$$\chi(\vec{r}, \vec{r}') = -\frac{1}{\pi} \text{Im} \int_{-\infty}^{\varepsilon_F} d\varepsilon G(\vec{r}, \vec{r}', \varepsilon) G(\vec{r}', \vec{r}, \varepsilon), \tag{9}$$

where J_{RKKY} is the RKKY coupling between side dots, J_i the antiferromagnetic coupling between the local state of dot $i (i = 2, 3)$ and the Kondo resonant state of embedded dot, and $\chi(\vec{r}, \vec{r}')$ the spin susceptibility of embedded dot($\vec{r}$ and $\vec{r}'$ are the positions of side dots with RKKY interaction). And $G(\vec{r}, \vec{r}', \varepsilon)$ is the Green function of embedded dot, which exhibit the Kondo resonant peak around Fermi level. J_i is given by

$$J_i = \frac{1}{2} t_{1i}^2 \left(\frac{1}{\varepsilon_F - \varepsilon_i} + \frac{1}{\varepsilon_F - \varepsilon_i} - \frac{1}{\varepsilon_F - \varepsilon_i - U_i} - \frac{1}{\varepsilon_F - \varepsilon_i - U_i} \right) .\tag{10}$$

Neglecting the position dependency, the RKKY coupling at zero temperature is given by

$$J_{RKKY} = -\frac{1}{\pi} J_2 J_3 \text{Im} \int_{-\infty}^{\varepsilon_F} d\varepsilon G(\varepsilon) G(\varepsilon). \tag{11}$$

Here $G(\varepsilon)$ can be written as

$$G(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_F + iT_k}, \tag{12}$$

and consequently the RKKY coupling yields to

$$J_{RKKY} \approx -\alpha \frac{J_2 J_3}{T_K}, \tag{13}$$

with $\alpha = O(1)$ a positive constant. J_i is

$$J_i = J = 4t^2/U \qquad (i = 2, 3) \tag{14}$$

at particle-hole symmetric point for $t_{12} = t_{13} = t$, $U_2 = U_3 = U$. The Kondo temperature T_K in embedded dot can be calculated from [27,28]

$$T_K = 0.182 U \sqrt{\overline{J}_{eff}(\Lambda)} \exp[-1/\overline{J}_{eff}(\Lambda)], \tag{15}$$

$$\overline{J}_{eff}(\Lambda) = \rho_0 J_K(g)/A_\Lambda, \tag{16}$$

$$A_\Lambda = \frac{1}{2} \ln \Lambda \left[\frac{1 + \Lambda^{-1}}{1 - \Lambda^{-1}} \right]. \tag{17}$$

In the "spin Kondo" regime, $\rho_0 J_K(g)$ is as follows [21]:

$$\rho_0 J_K(g) = \frac{8\Gamma}{\pi U} \sum_{m=0}^{\infty} \frac{|\langle 0|\widetilde{U}^{+}|m\rangle|^2}{1 - 2g^2/\omega_0 U + 2m\omega_0/U}, \tag{18}$$

$$|\langle 0|\widetilde{U}^{+}|m\rangle|^2 = \exp[-(g/\omega_0)^2](g/\omega_0)^{2m}/m!. \tag{19}$$

The above equations are valid for satisfying $2g^2/\omega_0 < U$(the conventional "spin Kondo" regime), i.e. the condition that the effective electron-electron interaction becomes repulsive. As seen in these equations, the Kondo temperature of embedded dot exhibits the raising tendency with the increase of electron-phonon coupling. If electron-phonon coupling is larger than the critical value, leading to $2g^2/\omega_0 > U$, "charge Kondo effect" appears instead of conventional Kondo effect. In this case, we cannot use them.

With taking both of direct exchange and indirect RKKY couplings into account, the effective exchange coupling between dot 2 and 3 is given by

$$J_{eff} = J' + J_{RKKY}, \tag{20}$$

where direct exchange coupling is

$$J' = \frac{4t'^2}{U}, \tag{21}$$

with $t_{23} = t'$. When $J_{eff} > 0$, the singlet state formed between dots 2 and