



Exact energy levels and eigenfunctions of an electron on a nanosphere under the influence of a radial magnetic field



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ABSTRACT

The exact energy levels and wave functions of an electron that is free to move on a nanosphere under the influence of a radial magnetic field have been determined. The wave functions are expressed in terms of Jacobi polynomials that are well defined and orthogonal and can be expressed using recurrence relations and series expansions. We also discuss the wave functions and energy levels in the presence of a very high magnetic field. Landau energy levels are shown for strong constant magnetic fields occurring on two-dimensional flat surfaces, if the radius is very large. The results are compared with those of previously published researches.

1. Introduction

Flat, two-dimensional systems can be created with electrons on liquid helium and solid state systems. Similarly, curved two-dimensional systems can be created by both on helium and solid state systems. In the first case, a multi-electron bubble resides in the liquid helium; the second example is a metallic nanoshell composite [1].

The electronic properties of curvilinear surfaces have attracted considerable attention since the discovery of carbon nanostructures. The motion of charged particles on a sphere under a constant magnetic field have been studied by numerous authors to discuss the fractional quantum Hall effect [2–5], weak magnetic field and strong magnetic field properties [6,7], Landau levels [8,9], and optical properties [10].

Goddard and Olive [11] gave a detailed description of the methods employed for creating radial magnetic fields (using Dirac magnetic monopoles and Dirac strings). The energy levels of the charged particle on a sphere under a radial magnetic field were investigated by Ralko and Truong in both classical and quantum mechanical regimes. They obtained solutions in Heun functions, which are a generalization of Gauss hypergeometric functions. In one of the coefficients of the Heun equations, they found a condition that led to the quantification of energy levels [12].

Tamm [13] solved the Schrödinger equation for the sphere in the presence of a magnetic monopole and found that the energy spectrum depends on the magnetic charge n as $E_{n,\ell} = \frac{\hbar^2}{2m^*R^2}[\ell(\ell+1) + |n|(\ell+1/2)]$ with m^* being the mass of the particle, R the sphere radius, and $\ell = 0, 1, 2, \dots$ the angular momentum quantum number. Avisha and Luck performed studies on tight-binding electronic spectra on graphs with the topology of a sphere [14].

They analyzed a one-electron spectrum as a function of the radial magnetic field produced by a magnetic charge located at the center of the sphere. Their analysis of the spectrum consisted of the five Platonic solids (tetrahedron, cube, octahedron, dodecahedron, and icosahedron), the C60 fullerene, and two families of polyhedra, the diamonds and the prisms.

While Haldane [2] and Ralko and Truong [12] determined the single-particle wave functions, they considered the magnetic vector for the radial magnetic field potential as $\mathbf{A} = -\frac{\hbar S}{eR} \cot \theta$, where the total magnetic flux $\hat{\phi}$ through the surface S in units of the flux quantum $\Phi_0 = h/e$ is an integer as required by Dirac's monopole quantization condition. In this study, the magnetic vector potential is determined in an efficient manner in Section 2 as done by Goddard and Olive [11]. We assume that there is no radial dependence of the movement of the electron on the sphere and that it depends on only two angle variables. After solving the time-independent Schrödinger equation, we found the exact energy eigenvalues and the wave functions of an electron on a two-dimensional spherical surface under a radial magnetic field. The eigenfunctions can be expressed in terms of Jacobi polynomials that have orthogonality and recurrence relations. In the absence of a magnetic field, we show that the eigenfunctions are reduced to Legendre polynomials. In the $p \rightarrow \infty$ limit, we show that the eigenfunctions are reduced to Laguerre polynomials, where p is the magnetic flux in units of the flux quantum $\Phi_0 = h/e$. Furthermore, we discuss the sphere radius limit $R \rightarrow \infty$, in which the sphere surface can be taken as a flat surface, and energy levels are shown to develop into two-dimensional Landau energy levels. In the last section, the results are discussed.

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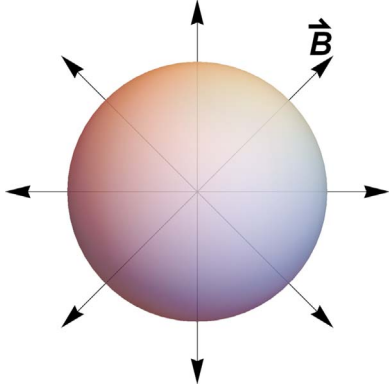


Fig. 1. A nanosphere under the influence of a radial magnetic field created by the Dirac string or Dirac magnetic monopole.

2. Nanosphere under the influence of a radial magnetic field

Electrons on a sphere are strongly bound perpendicular to the surface, while they move freely in directions parallel to the surface. On a sphere, this means that the full (three-dimensional) wave function describing such electrons should be factorizable into a function that depends only on the angles and a function that depends only on the radial distance. The system can be considered two-dimensional if all the electrons have the same radial dependence of their wave function and if the energy required to change the radial mode is much larger than the other relevant energy scales involved.

A nanosphere under the influence of a radial magnetic field created by the Dirac string or Dirac magnetic monopole [11] is shown in Fig. 1. In the absence of magnetic field, electrons that can move freely on the sphere are in single-particle angular momentum states. For a rigid sphere of radius R , the energy of a single electron confined to the surface is called the rigid rotator energy $E_\ell = \frac{\hbar^2}{2m^*R^2}\ell(\ell+1)$, where ℓ is the angular momentum quantum number, m^* the mass of the electron, and $\hbar$ is Planck's constant.

The Schrödinger equation of an electron on a metallic nanosphere under a radial magnetic field is

$$\frac{1}{2m^*}[-i\hbar\nabla + e\mathbf{A}]^2\psi = E\psi \quad (1)$$

where $\mathbf{A}$ is the vector potential on the sphere resulting from the radial magnetic field and m^* and e are the effective mass and charge of the electron, respectively.

Using spherical polar coordinates (r, θ, ϕ) , we expect to be able to find a vector potential $\mathbf{A}(\theta) = A(\theta)\hat{\phi}$, with $\hat{\phi}$ being a unit vector in the ϕ direction, by symmetry. The magnetic flux through a circle C , corresponding to fixed values of R and θ and ϕ ranging over the values 0 to 2π , is given by the solid angle subtended by C at the origin multiplied by $\frac{1}{4\pi}\int \mathbf{B}\cdot d\mathbf{S}$, namely, $\frac{(1-\cos\theta)}{2}\int \mathbf{B}\cdot d\mathbf{S}$. The total magnetic flux from the sphere surface is $\Phi = \int \mathbf{B}\cdot d\mathbf{S}$ [11]. Consequently,

$$\frac{(1-\cos\theta)}{2}\Phi = 2\pi A(\theta)R\sin\theta \quad (2)$$

and the vector potential is

$$\mathbf{A}(\theta) = \frac{\Phi}{4\pi R} \frac{(1-\cos\theta)}{\sin\theta} \hat{\phi} \quad (3)$$

If we insert the vector potential in Eq. (3) into Eq. (1) and look for the eigenfunctions in the form $\psi(\theta, \phi) = T(\theta)e^{im\phi}$, then

$$-\frac{\hbar^2}{2m^*R^2}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dT}{d\theta}\right) - \frac{m^2}{\sin^2\theta} - mp\frac{1-\cos\theta}{\sin^2\theta} - \frac{p^2(1-\cos\theta)^2}{4\sin^2\theta}\right]T(\theta) = ET(\theta) \quad (4)$$

where m is magnetic quantum number, $(m - \frac{p}{2})\hbar$ is the eigenvalue of

L_z , $\Phi_0 = \frac{h}{e}$ is the magnetic flux quantum, and $\Phi/\Phi_0 = p$ denotes the magnetic flux in units of the flux quantum Φ_0 . Defining a dimensionless form of energy $\epsilon = \frac{2m^*R^2}{\hbar^2}E$ and a change of variable $\mu = \cos\theta$, we can write

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dT(\mu)}{d\mu}\right] + \left[\epsilon - \frac{m^2}{1-\mu^2} - mp\frac{1-\mu}{1-\mu^2} - \frac{p^2(1-\mu)^2}{4(1-\mu^2)}\right]T(\mu) = 0 \quad (5)$$

By taking into account the singular points of the differential equation in Eq. (5), we can propose the solution as follows:

$$T(\mu) = (1-\mu)^{\frac{|m|}{2}}(1+\mu)^{\frac{|p+m|}{2}}P(\mu) \quad (6)$$

After substituting Eq. (6) into Eq. (5), we find

$$(1-\mu^2)\frac{d^2P(\mu)}{d\mu^2} + \{[p+m] - |m| - (|p+m| + |m| + 2)\mu\}\frac{dP(\mu)}{d\mu} + \left\{\epsilon - \frac{|p+m|(|m|+1) + |m|(p+m)m}{2}\right\}P(\mu) = 0 \quad (7)$$

Eq. (7) is a Jacobi differential equation and its solutions are Jacobi polynomials, $P_n^{(|m|, |p+m|)}(\mu)$ with integer quantum numbers $n = 0, 1, 2, \dots$, where m can take $-n \leq m \leq n+p$ integer values with angular momentum quantum numbers $\ell = n + \frac{p}{2}$ [11]. To find the energy eigenvalues, we use the dimensionless form of energy identity $\epsilon = n(n+1) + |p+m|(n + \frac{|m|+1}{2}) + |m|(n + \frac{1}{2}) + (p+m)\frac{m}{2}$ to get

$$E_{n,p,m} = \frac{\hbar^2}{2m^*R^2}\left[n(n+1) + |p+m|\left(n + \frac{|m|+1}{2}\right) + |m|\left(n + \frac{1}{2}\right) + (p+m)\frac{m}{2}\right] \quad (8)$$

This energy eigenvalue is the energy eigenvalue of an electron that freely moves on the surface of a sphere under a radial magnetic field. The total wave function is

$$\psi(\mu, \phi) = N_n^{p,m}(1-\mu)^{\frac{|m|}{2}}(1+\mu)^{\frac{|p+m|}{2}}P_n^{(|m|, |p+m|)}(\mu)e^{im\phi} \quad (9)$$

From the orthogonality property of Jacobi polynomials, the normalization constant $N_n^{p,m}$ is given as

$$N_n^{p,m} = \left[n! \frac{(2n+|p+m|+|m|+1)\Gamma(n+|p+m|+|m|+1)}{\pi 2^{|p+m|+|m|+2}\Gamma(n+|m|+1)\Gamma(n+|p+m|+1)}\right]^{1/2} \quad (10)$$

Where $\Gamma(z)$ is the Gamma function. In the absence of a magnetic field ($p = 0$ and $\ell = n$) in Eq. (5), the wave function is reduced to

$\psi(\mu, \phi) = \left(\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}\right)^{1/2} P_\ell^m(\mu)e^{im\phi}$, where $P_\ell^m(\mu)$ are the associated Legendre polynomials and the energy is reduced to $2\ell+1$ times the degenerate rigid rotator energy $E_\ell = \frac{\hbar^2}{2m^*R^2}\ell(\ell+1)$.

Now, we will analyze the $p \rightarrow \infty$ ($|p+m| \rightarrow \infty$) limit. If we change the variable μ to $x = \frac{|p+m|}{2}(1-\mu)$, the functions $\left(P_n^{(|m|, |p+m|)}(\mu), (1-\mu)^{\frac{|m|}{2}}, \text{ and } (1+\mu)^{\frac{|p+m|}{2}}\right)$ in Eq. (9) in this limit become

$\lim_{|p+m| \rightarrow \infty} P_n^{(|m|, |p+m|)}\left(1 - \frac{2x}{|p+m|}\right) = L_n^{|m|}(x)$ [15], where $L_n^{|m|}(x)$ are the

associated Laguerre polynomials, $(1-\mu)^{\frac{|m|}{2}} = \left(\frac{2}{|p+m|}\right)^{\frac{|m|}{2}} x^{\frac{|m|}{2}}$, and

$\lim_{|p+m| \rightarrow \infty} (1+\mu)^{\frac{|p+m|}{2}} \approx 2^{\frac{|p+m|}{2}} e^{-\frac{x}{2}}$.

Then the wave function becomes

$$\psi(x, \phi) = N_{n,m} e^{-\frac{x}{2}} x^{\frac{|m|}{2}} L_n^{|m|}(x) e^{im\phi} \quad (11)$$

The normalization constant $N_{n,m}$ is found from the orthogonality property of associated Laguerre polynomials, $N_{n,m} = \left(\frac{n!}{2\pi(n+|m|)!}\right)^{\frac{1}{2}}$. If we take $p \gg n$ and $p \gg |m|$, the energy eigenvalues in this limit ($p \rightarrow \infty$) from Eq. (8) are Landau energy levels

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