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Topological phase transition in the field induced Kitaev model on the Kagome lattice

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ABSTRACT

We investigate the low-energy spectral properties of the Kitaev model on the Kagome lattice, which is a quantum spin model for the aim of fault-tolerant quantum computation, in the presence of a uniform magnetic field in the x- and z-directions. We explore the low-energy physics of the Kitaev model in the x- and z-magnetic fields, separately and establish a quasi-particle picture for anyonic excitations. Our study is based on the high-order series expansion of the low- and high field limits of the problem by means of perturbative continuous unitary transformations. We further show that the Kitaev model in the x-field is mapped to the Ising transverse field (ITF) model on the triangular lattice while, the system is mapped to another ITF model on the honeycomb lattice in the presence of the z-magnetic field. Additionally, we investigate the phase transitions of the model for the two cases and find that the topological phase of the Kitaev model breaks down to the polarized phase in either x- or z-directions by a second-order quantum phase transition in the 3D Ising universality class. We further detect dispersive bound states in high-field limits of the model for both cases of the magnetic field. The overall results further indicate that the Kitaev model on the Kagome lattice has the different stability as the toric code on the square lattice, while perturbed by magnetic fields.

1. Introduction

In condensed matter physics, conventional phases of matter are characterized by their atomic structure or internal order. The Ginzburg-Landau symmetry-breaking theory associates this internal order to the symmetry and attempts to present a general theory to classify all phases of matter based on their symmetries [1]. Changing the local order by tuning a physical quantity such as temperature or magnetic field, would results in spontaneous breaking of symmetry at a critical point and the material undergoes a phase transition. For years, it was believed that the Landau theory is the standard model for characterizing every possible phases of matter until 1982 that the discovery of the Fractional Quantum Hall Liquid (FHQ) by Tsui et al. [2] came as a big surprise. The order in this new phase was not related to any kind of symmetry. Therefore the Landau's theory was not capable of describing this new phase based on the symmetries. Novel properties of the FHQs such as edge states and the degeneracy of the ground state which was related to the topology of the space, led physicists to introduce a new kind of order by resorting to topological quantum numbers which characterized this new phase. The new order was therefore distinguished as *topological order* [3,4].

Later on, more examples of systems with topological order emerged in superconducting states [5,6], short range resonating valence bonds [7–10] and quantum spin models [11–19] as well as several proposals for experimental implementation of spin systems on optical lattices [20–22]. Highly entangled states and robustness of electronic states in topologically ordered systems [23,24] further attracted scientists of quantum information field to define non-local quantum bits on the topological degrees of freedom and protect information from decoherence [25–27]. The central idea behind this non-locality was to distribute the quantum entanglement between many different particles in such a way that it can not be destroyed by local perturbations. The first exactly solvable quantum spin model with topological protection was the toric code [25] by Kitaev which was designed on a four valent lattice wrapped around a torus with arbitrary genus and used the topologically degenerate ground state of the system to describe a robust quantum memory. The ground state of the system was separated from the excited states by a gap and the excitations were anyonic quasi-particles with exotic statistics. Most interestingly, braiding of the non-Abelian anyons could be used to generate unitary quantum gates, making toric code as a rich test ground for fault tolerant quantum computation [28–30].

Aside from interesting characteristics of the quantum codes for fault-tolerant quantum computation, there are many questions about

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other physical properties of such topological phases, from many-body or rather condensed matter physics aspects of view. Investigating the low-energy effective theory of toric code on the square lattice [31,32] and its phase transitions in magnetic field [33–37] and several experimental proposals for detection of anyonic excitations on optical lattices [38,39], have been the subjects of different studies to answer some of the questions about this model. However, there are still lots of unanswered question that should be the motivation for further studies.

The aim of the present work is to investigate the low-energy spectral properties of Kitaev model on the Kagome lattice in the presence of uniform magnetic fields in the x- and z-direction and shed light on the physics of the perturbed Kitaev model on the Kagome lattice. We study the low-energy spectral properties of the Kitaev model in two different magnetic fields by using high-order series expansion of the low- and high-field limits of the problem and present a quasi-particle (QP) picture for the anyonic excitations. Additionally, we provide exact mappings for the problem and show while the Kitaev model in the x-field is mapped to ITF model on the triangular lattice, the model is mapped to another ITF model on the honeycomb lattice in the presence of the z-field. We further investigate the quantum phase transitions and dynamics of the model by setting up a QP picture. As we shall see, the high-field limits of the problem turns out to have very interesting characteristics such as emergence of dispersive bound states in the system. Finally, we compare our results with those of the toric code on the square lattice.

The paper is organized as follows: In Section 2, we review the structure of the Kitaev model on the Kagome lattice and the relevant properties of the system which are mostly used in the rest of the paper. The perturbed models with x- and z-magnetic fields are introduced in Section 3. We further describe the mapping of the model to two different ITF model in Section 4 and we present the high-order series expansions of the low- and high-field limits for both magnetic fields in the x- and z-direction in Section 5. The next Section 6 is devoted to the discussion on the phase transitions out of the topological phase of the Kitaev model. Eventually, the paper ends with conclusion in Section 7.

2. Model

Consider a two dimensional Kagome lattice where spin $\frac{1}{2}$ particles are placed on its vertices (Fig. 1) and the interactions between the spins are tuned with the following Hamiltonian [40]

$$H = -J_v \sum_v A_v - J_p \sum_p B_p \tag{1}$$

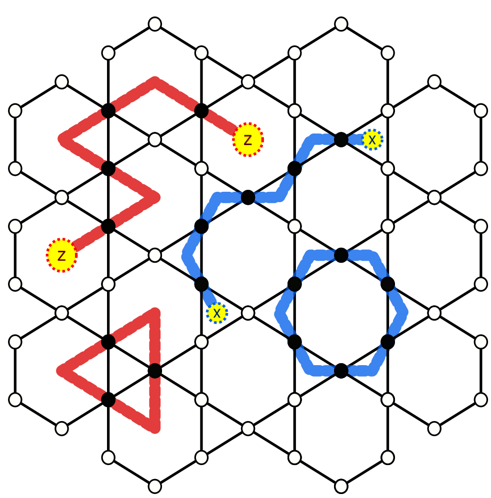

Fig. 1. A scheme of two-dimensional Kagome lattice. Empty and filled circles represent spin $\frac{1}{2}$ with different directions. X and Z quasi-particle excitations appear at the end points of open strings.

where, $A_v = \prod_{i\epsilon v} \sigma_i^x$ is vertex operator acting on the spins around a triangle, $B_p = \prod_{j\epsilon p} \sigma_j^z$ is plaquette operator acting on hexagons and $\sigma^{x(z)}$ are standard Pauli matrices. The vertex and plaquette operators share nothing or even number of sites and therefore commute with each other, $[A_v, B_p] = 0$, leading to the exact solvability of the Hamiltonian (1).

The vertex and plaquette operators squares to identity, $(A_v)^2 = (B_p)^2 = \mathbf{1}$, and their eigenvalues are therefore given by ± 1. Setting $J_v, J_p > 0$, ground state of the Hamiltonian (1) correspond to a state where all eigenvalues of vertex and plaquette operators are equal to $+1$ with ground state energy $E_0 = -J_v N_v - J_p N_p$ which for a Kagome lattice with $3N$ sites (N plaquette, $2N$ vertex) is equals to $E_0 = -2NJ_v - NJ_p$. Excitations of the model further corresponds to -1 eigenvalues of the vertex and plaquette operators. The system is therefore gapped. Hereafter we set $J_v = J_p = J$, therefore the first excited state of the system corresponds to one of vertices or plaquettes being violated (with -1 eigenvalue) which has $2JN_{v,p}$ energy cost.

Beside the vertex and plaquette operators, there are other entities on lattice, called string operators which are generalization of the vertex and plaquette operators. The strings can be open or closed and string operators are constructed as the product of Pauli spins on the string (see Fig. 1). Closed strings always commute with the vertex and plaquette operators. However, if they are open, they anti-commute with vertex and plaquette operators at their end points and create a pair of excitations, the so-called anyonic charges and fluxes on the corresponding vertex and plaquettes. These charges and fluxes are deconfined quasiparticles (QP) of the model which are hardcore bosons and has mutual semionic statistics [41].

Wrapping the system around a torus, a new class of closed strings i.e. the global strings appear for every homology class of the torus which are not the product of plaquette and vertex operators but still commute with them. As will be seen shortly, these global strings are responsible for the topological degeneracy of the ground state of the system.

Starting from a polarized spin background, the very general form of the ground state of the system is constructed as follows:

$$|\psi_{gs}\rangle = \mathcal{N} \prod_v (1 + A_v)|0\rangle^{\otimes N} \tag{2}$$

where $\mathcal{N}$ is a normalization constant and $|0\rangle$ is the eigenstate of the σ_z Pauli operator with $+1$ eigenvalue. This state is a superposition of the strongly fluctuating closed strings or a string condensate. Such a configuration can be visualized by the product of all vertex operators acting on the polarized spin background of the system. This is a feature which is found in systems with topological order [24].

As we have previously outlined, the global strings of the manifold commute with the vertex and plaquette operators while they are not the product of those operators. As a result, any string state which is generated from the action of global strings on the ground state (2) is a topologically degenerate ground state of the system. The Kitaev model has two independent global loops of each homology on the Kagome lattice wrapped around a torus of g=1 which leads to a 4-fold degenerate ground-space (see Fig. 2).

3. Kitaev model in magnetic field

In order to reveal the physics of the Kitaev model and to investigate its spectral properties in a magnetic field, we add two magnetic field terms in different directions to the Hamiltonian (1). However, we investigate each case separately in the small- and large-field limits. The Hamiltonian of the Kitaev model in the presence of magnetic fields reads:

$$H = -J\sum_v A_v - J\sum_p B_p - h_x \sum_i \sigma_i^x - h_z \sum_i \sigma_i^z \tag{3}$$

where h_x, h_z represent magnetic field strength in the x- and z-direc-