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Nonlocal excitation and potential instability of embedded slender and stocky single-walled carbon nanotubes under harmonically vibrated matrix

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ABSTRACT

Until now various aspects of vibrations of single-walled carbon nanotubes (SWCNTs) have been explored; however, their dynamics and possible instabilities because of the excitation of matrix have not been addressed methodically. By considering a harmonic transverse excitation, the explicit expressions of elastic fields are obtained based on the nonlocal Rayleigh, Timoshenko, and higher-order beam models. The roles of the nonlocality, slenderness ratio, amplitude and frequency of matrix excitation and interactional behavior of the embedded nanotube on the dynamic transverse displacements of SWCNTs are comprehensively displayed. The capabilities of the Rayleigh model as well as the Timoshenko model in capturing the deflection of the nanostructure based on the higher-order beam theory are also explained in some detail. The nonlocal elastodynamic fields of the nanostructure in the resonance state as well as the critical values of lateral and rotational stiffness of the matrix are also introduced and the influences of crucial factors on such parameters are explained and discussed carefully.

1. Introduction

There exists a high interest in exploiting carbon nanotubes (CNTs) as the reinforcing phase in nanocomposites [1–5]. This fact is mainly related to their superior toughness, stiffness, chemical, and physical properties [6–9] in which suggests them as one of the most strong materials. For practical purposes, mechanical capability of the nanocomposite in transverse vibration should be realized carefully; however, before proceeding on this complex mechanism, we should understand the nature of the resulting vibrations in these tiny fibers because of their interactions with the surrounding elastic matrix. To this end, herein, we perform an interesting study to determine the nonlocal dynamic response of single-walled carbon nanotubes (SWCNTs) by considering their dynamic interactions with the matrix. In brief, suitable nonlocal models are established and the exact elastodynamic fields within such beam-like small elements are evaluated. Surely, these investigations provide a solid base for upcoming explorations on nanocomposites reinforced by vertically aligned SWCNTs, double-walled carbon nanotubes (DWCNTs) or even multi-walled carbon nanotubes (MWCNTs).

Commonly, application of atomistic-based models in dynamic analysis of nanostructures is time-consuming and it takes considerable labor costs. This issue becomes highlighted when the nanostructure is excited by an external force, and forced vibration analysis is of interest. To overcome these shortcomings of atomic simulations, several advanced continuum-based theories have been elaborated through the last century. One of the most well suited models is that developed by Eringen [10–12], commonly called *nonlocal continuum-field theory*. Simply, this model expresses that the state of strain or stress at any point of the nanobody is not only affected by the strain or stress of that point, but also by the strains and stresses of the adjacent points. On the basis of this theory, the nonlocal stresses are expressed by: $\sigma_{ij}^{nl}(\mathbf{x}) = \int_{\Omega} \Lambda(|\mathbf{x}-\mathbf{x}'|, \frac{e_0 a}{l})\sigma_{ij}^{l}(\mathbf{x}')d\Omega(\mathbf{x}')$ where σ_{ij}^{l} denote the local stress components, Ω is the whole spatial domain, Λ is an appropriate kernel function, $e_0 a$ represents the small-scale parameter, l is an important characteristic of the nanostructure, and $|\mathbf{x}-\mathbf{x}'|$ is the distance between points with coordinates $\mathbf{x}$ and $\mathbf{x}'$. The kernels are particular bounded functions that satisfy the completeness condition as well [13]. Application of the integro form of nonlocal stresses for dynamic analysis of nanostructures results in complex integro-partial differential equations that finding their solutions is a cumbersome task. In order to get rid of this complexity, Eringen [12] proposed a more simple constitutive relation for mechanical analysis of elastic solids, namely: $\sigma_{ij}^{nl} - (e_0 a)^2 \nabla^2 \sigma_{ij}^{nl} = \sigma_{ij}^{l}$ where ∇ is the nabla symbol.

To date, various aspects of vibrations of CNTs have been cultivated by implementing the nonlocal elasticity theory of Eringen. For instance, free transverse vibrations of SWCNTs [14–22], DWCNTs [23–27], doubly orthogonal SWCNTs [28,29], MWCNTs [30,31], membranes and jungles of vertically aligned SWCNTs [32,33] have been investigated. The capability of SWCNTs in transferring nonlocal sound

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waves [34–37], local and nonlocal vibrations of SWCNTs and MWCNTs in the presence of magnetic fields [38–42], and nonlocal-nano-mechanical sensing of nano-objects by SWCNTs [43–47] have been also of concern of applied mechanics researchers in recent years. Additionally, strain gradient theory has been implemented for vibrational analysis of nanotubes and nano-scaled beams [48,49].

Among all performed studies, the effect of vibration of surrounding elastic medium on vibro-elastic analysis of SWCNTs has not been researched systematically. From dynamical point of view, this is classified as *forced vibration problems* and the atomic models cannot be easily employed due to their high computational and labor costs. In return, appropriate nonlocal beam-like models could be exploited to capture of our interested data. Given the importance of the subject in mechanical analysis of nanocomposites consist of CNTs as the reinforcing phase, the authors are encouraged to propose nonlocal analytical models to examine the problem for both slender and stocky SWCNTs. Further, the influences of the geometrical and mechanical data of both SWCNTs and matrix as well as frequency of harmonic transverse waves within the matrix phase on the resulted deflections of the nanostructure and its possible instability are explored methodically. The main features of the present work are determination of the nonlocality and shear roles on the deformation of such tiny structures due to excitation of the matrix. Additionally, the nonlocal resonance deformations of the nanostructure as well as the critical values of lateral stiffness and rotational stiffness are displayed and the effects of influential factors on these parameters are investigated in some detail.

2. Assumptions and description of the nanomechanical problem

Consider a SWCNT of length l_b and radius r_m that has been confined by an elastic matrix as demonstrated in Fig. 1(a). The SWCNT is under transverse excitation of the matrix along the z-direction as: $w_g(t) = a_g \sin(\varpi t)$ where a_g and ϖ are the amplitude and the frequency of the matrix excitation. For mathematical modeling of a SWCNT, the equivalent continuum structure (ECS) pertinent to the nanostructure is exploited on the basis of works of Batra and Gupta [50] as well as Gupta and Batra [51]. The ECS is a hollow circular cylindrical body whose length and mean radius are the same as the length and radius of the parent SWCNT, respectively. The wall's thickness of the ECS (t_b) is set equal to 0.34 nm in all performed calculations. The Cartesian coordinate system has been attached to the left support such that the x-axis passes through the revolutionary axis of the SWCNT, and the z-axis is pointed towards the gravitational acceleration. The dynamic interactions of the nanotube with the nearby matrix is simulated by continuous lateral and rotational elastic springs whose stiffness are k_t and k_r, respectively (see Fig. 1(b)).

Herein, the roles of internal damping of both nanotube and matrix and external damping (which is mainly resulted from the friction and adhesion of the SWCNT with the matrix) in dynamic analysis of the problem have not been considered. To the best of our knowledge, there exist no inclusive works on the exact mechanism and experiment of internal damping of CNTs, and more investigations are still required to explain this crucial phenomenon. However, there are some nonlocal studies regarding nonlocal modeling of internal damping in nanobeams [52–55]. Concerning external damping, the friction between CNTs and their adjacent composites has been displayed and discussed in several experimental works [56–58]. In order to incorporate such effects into the formulations, the previously developed models on the influence of interlayer damping on vibrations of double-macro-beam systems [59,60] would be very helpful. To construct more sophisticated and rational models, these particular effects should be included in the governing equations appropriately. This brief explanation displays several hot topics on forced vibration of embedded SWCNTs that should be seriously paid attention to by the applied nanomechanics community.

3. Vibration of SWCNTs under matrix excitation via nonlocal slender and shear deformable beams

In the following parts, the nonlocal equations of motion of the excited nanostructure, which has been in contact with the elastic matrix, are derived according to the nonlocal Rayleigh beam model (NRBM), nonlocal Timoshenko beam model (NTBM), and nonlocal higher-order beam model (NHOBM) and then, an analytical solution is proposed to solve the resulting boundary value problems.

3.1. Application of the NRBM to excited SWCNTs

The only source of vibration within the nanostructure is resulted from the transverse matrix excitation. On the basis of the NRBM, the transverse vibration of the SWCNT is given by:

$$\rho_b(A_b \ddot{w}_t{}^R - I_b \ddot{w}^R_{t,xx}) - (M_b^{nl})^R_{,xx} + k_t w_t^R - k_r w^R_{t,xx} = K_t w_g, \tag{1}$$

where the overdot sign represents the derivative w.r.t time, the subscript $,x$ denotes the first derivative w.r.t x, ρ_b is the density of the ECS, A_b and I_b are the area and the moment inertia of the cross-section of the ECS, $w^R(x, t)$ is its pure dynamic deflection field, $(M_b^{nl})^R$ is the nonlocal bending moment field of the Rayleigh model, and

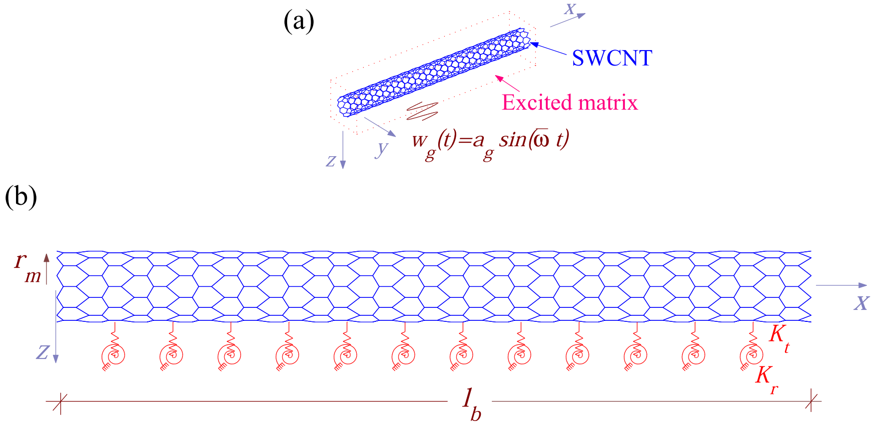

Fig. 1. (a) A SWCNT in a harmonically excited matrix; (b) Modeling of the nearby matrix via elastic transverse and rotational springs.