Physica B 509 (2017) 94–99

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Andreev reflection properties in a parallel mesoscopic circuit with Majorana bound states

Jin-Tao Mu[a], Yu Han[a], Wei-Jiang Gong[b,*]

[a] Physics Department, Liaoning University, Shenyang 110036, China
[b] College of Sciences, Northeastern University, Shenyang 110819, China

ARTICLE INFO

Keywords:
Andreev reflection
Majorana bound state
Quantum dot
Magnetic flux

ABSTRACT

We investigate the Andreev reflection in a parallel mesoscopic circuit with Majorana bound states (MBSs). It is found that in such a structure, the Andreev current can be manipulated in a highly efficient way, by the adjustment of bias voltage, dot levels, inter-MBS coupling, and the applied magnetic flux. Besides, the dot-MBS coupling manner is an important factor to modulate the Andreev current, because it influences the period of the conductance oscillation. By discussing the underlying quantum interference mechanism, the Andreev-reflection property is explained in detail. We believe that all the results can assist to understand the nontrivial role of the MBSs in driving the Andreev reflection.

1. Introduction

Heterostructures with quantum dots (QDs) coupled to superconducting and normal leads are the well-known systems to study the interplay between the superconducting correlations and mesoscopic electronic transport [1–10]. In the structure of one normal metallic lead coupled to the superconductor via QDs, the Andreev reflection has an opportunity to take place. In such a process, an incident electron from a normal metal can be converted into a Cooper pair in the superconductor when its energy is less than the superconductor energy gap Δ, and thus a hole with opposite spin and velocity reflects back. Also, in multi-terminal systems, the two electrons in a Cooper pair from the same lead or different leads are distinguished as the normal and crossed Andreev reflection ones [11,12]. During the past years, there is a growing attention in studying the transport properties of the superconductor coupled mesoscopic hybrid systems for both the fundamental interest and potential applications in nanoelectronics. The transport properties in various configurations of metal-QD-superconductor have been investigated [13–16].

In recent years, topological superconductor (TS) has become one important concern in the field of mesoscopic physics due to the presence of Majorana mode at its boundary [17–21]. It is known that Majorana bound states (MBSs) have been realized at the ends of a one dimensional p-wave superconductor for which the proposed system is a semiconductor nanowire with Rashba spin–orbit interaction to which both a magnetic field and proximity-induced s-wave pairing are added [21]. Following the fabrication of the MBSs in solid states, the MBS-assisted transport properties in the mesoscopic circuit have become one of the hot topics in the field of mesoscopic physics. And some interesting results have been reported. For instance, when a pair of MBSs couple to the two leads of one circuit, the nonlocality of the MBSs was observed due to the occurrence of the crossed Andreev reflection [22]. In the junction between a normal metal and a chain of coupled MBSs, the Andreev reflection shows odd-even effects, i.e., when the MBS number is odd, the zero-bias conductance peak is of height $2e^2/h$, whereas it is equal to zero otherwise [23]. Moreover, the coupling between the MBSs and regular bound states has been found to bring about new transport properties of a mesoscopic circuit [24]. When the QD is noninteracting and in the resonant-tunneling regime, the MBS influences the conductance through the QD by inducing the sharp decrease of the conductance by a factor of $\frac{1}{2}$ [25,26]. If the QD is in the Kondo regime, the QD–MBS coupling reduces the unitary-limit value of the linear conductance by exactly a factor $\frac{3}{4}$ [27].

One of the transport characteristics of the mesoscopic circuit is its dependence on the quantum interference effect which is related to the circuit geometry. For a quantum ring, the current flow will show oscillation behavior by changing the applied local magnetic flux. Such a result is exactly the Aharonov–Bohm (AB) effect [28]. Consequently, the AB effect drives various interesting transport results in the QD structures, by cooperating with other system parameters, such as the QD levels and intradot Coulomb interaction. Therefore, it can be anticipated that when QDs couple with MBSs to form quantum ring, the quantum transport result will be more complicated and interesting, due to the coexistence of multiple transport behaviors governed by the

* Corresponding author.
E-mail address: gwj@mail.neu.edu.cn (W.-J. Gong).

http://dx.doi.org/10.1016/j.physb.2017.01.007
Received 3 September 2016; Received in revised form 7 December 2016; Accepted 8 January 2017
Available online 10 January 2017
0921-4526/

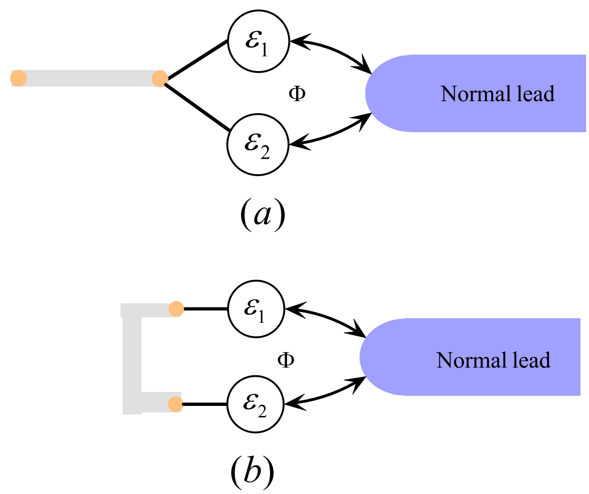

Fig. 1. Sketches of a parallel mesoscopic ring with coupled MBSs, in which one QD is embedded in each arm. One local magnetic flux is introduced to adjust the quantum interference. (a) Two QDs couple to one MBS. (b) Two QDs connect with two MBSs, respectively.

quantum interference. In the present work we aim to investigate the Andreev reflection in a parallel mesoscopic circuit with MBSs. After calculation, we find that the current induced by the Andreev reflection can be manipulated in a highly-efficient way, by adjusting the relevant quantities, e.g., the bias voltage, QD levels, inter-MBS coupling, and the applied magnetic flux. In addition, the coupling between the MBSs and QDs makes an important contribution to modification of the Andreev current. The obtained results can be helpful in understanding the properties of the MBS-contributed Andreev reflection.

2. Model

The parallel mesoscopic circuit with coupled MBSs is illustrated in Fig. 1. Thanks to the advance of the nanotechnology, this structure can be fabricated in experiment. The QDs can be fabricated by applying gate voltages on the two-dimensional electron gas [29,30]. Besides, when a semiconductor nanowire with strong Rashba interaction is subjected to a strong magnetic field $\mathbf{B}$ and adheres to a proximity-induced s-wave superconductivity, a pair of MBSs can form at the ends of the nanowire [26].

The Hamiltonian that describes the electron motion in the structure of Fig. 1 can be written as

$$H = H_{NM} + H_M + H_D + H_T. \tag{1}$$

The first term is the Hamiltonian for the normal metallic lead, which takes the form as $H_{NM} = \sum_k \varepsilon_k c_k^\dagger c_k$. $c_k^\dagger$ (c_k) is an operator to create (annihilate) an electron of the continuous state $|k\rangle$ in the normal metallic lead. ε_k is the corresponding single-particle energy. H_M represents the Hamiltonian for the MBSs

$$H_M = i\epsilon_M \eta_1 \eta_2. \tag{2}$$

This term in Eq. (2), the low-energy effective Hamiltonian for the MBSs, describes the paired MBSs generated at the ends of the nanowire and coupled to each other by an energy $\epsilon_M \sim e^{-l/\xi}$, with l the wire length and ξ the superconducting coherent length. In Ref. [22], the relation between ϵ_M and l and ξ has been demonstrated, which is helpful for the relevant experiment.

H_D, the QD Hamiltonian, can be written as $H_D = \sum_j \varepsilon_j d_j^\dagger d_j$. $d_j^\dagger$ (d_j) is the creation (annihilation) operator of electron in QD-j, and ε_j denotes the electron level in the corresponding QD. Finally, H_T represents the coupling between the QDs and lead(MBSs), and it can be expressed as

$$H_T = \sum_k t_j c_k^\dagger d_j + (\lambda_{jl} d_j - \lambda_{jl}^* d_j^\dagger)\eta_l + h.c.. \tag{3}$$

λ_{jl} is the coupling coefficient between the QDs and MBSs. t_j represents the coupling between QD-j and the lead. In this work, we would like to pay attention to two typical models, i.e., Model I where $\lambda_{11} = \lambda_1$ and $\lambda_{21} = \lambda_2$ with $\lambda_{12} = \lambda_{22} = 0$ and Model II where $\lambda_{11} = \lambda_1$ and $\lambda_{22} = \lambda_2$ with $\lambda_{21} = \lambda_{12} = 0$, as shown in Fig. 1(a)–(b).

With the help of the nonequilibrium Green function technique, the Andreev current in the normal metallic lead can be given by [31]

$$J = \frac{e}{h}\int d\omega T_{eh}(\omega)[f(\omega - eV) - f(\omega + eV)]. \tag{4}$$

$f(\omega)$ is the Fermi distribution in the normal metallic lead. $T_{eh}(\omega) = \mathrm{Tr}[\Gamma_E \mathbf{G} \Gamma_H \mathbf{G}^\dagger]$ is the Andreev reflection ability, where $\mathbf{G}$ is the related Green function. $\Gamma_{E(H)}$ are the coupling matrix between the electron (hole) bound state and the lead, respectively. With the help of equation of motion method, the matrix form of the Green function can be obtained, i.e.,

$$\mathbf{G}(\omega) = \begin{bmatrix} g_{1e}(\omega)^{-1} & \frac{i}{2}\Gamma_{12}^e & 0 & 0 & \lambda_{11}^* & \lambda_{12}^* \\ \frac{i}{2}\Gamma_{21}^e & g_{2e}(\omega)^{-1} & 0 & 0 & \lambda_{21}^* & \lambda_{22}^* \\ 0 & 0 & g_{1h}(\omega)^{-1} & \frac{i}{2}\Gamma_{12}^h & -\lambda_{11} & -\lambda_{12} \\ 0 & 0 & \frac{i}{2}\Gamma_{21}^h & g_{2h}(\omega)^{-1} & -\lambda_{21} & -\lambda_{22} \\ \lambda_{11} & \lambda_{21} & -\lambda_{11}^* & -\lambda_{21}^* & g_1(\omega)^{-1} & -i\epsilon_M \\ \lambda_{12} & \lambda_{22} & -\lambda_{12}^* & -\lambda_{22}^* & i\epsilon_M & g_2(\omega)^{-1} \end{bmatrix}^{-1}, \tag{5}$$

where $g_{je}(\omega) = [\omega - \varepsilon_j + \frac{i}{2}\Gamma_{jj}^e]^{-1}$, $g_{jh}(\omega) = [\omega + \varepsilon_j + \frac{i}{2}\Gamma_{jj}^h]^{-1}$, and $g_l(\omega) = [\omega + i0^+]^{-1}$. Γ^e and Γ^h are the selfenergies induced by the QD-lead coupling, where are given by $\Gamma_{jl}^e = 2\pi t_j t_l^* \rho_e$ and $\Gamma_{jl}^h = 2\pi t_j^* t_l \rho_h$. Within the wide-band limit approximation, ρ_e can be considered to be identical with ρ_h, which will leads to the result of $\Gamma_{jj}^e = \Gamma_{jj}^h$. In this work, we are only interested in the case of symmetric QD-lead coupling, i.e., $|t_j| = t_0$. Thus, the matrix forms of the Γ_E and Γ_H can be simplified as

$$\Gamma_E = \Gamma_0 \begin{bmatrix} 1 & e^{i\phi} & 0 & 0 & 0 & 0 \\ e^{-i\phi} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

and

$$\Gamma_H = \Gamma_0 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & e^{i\phi} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

with $\Gamma_0 = 2\pi t_0^2 \rho_{e(h)}$. In the above equations, ϕ is the magnetic phase factor induced by the finite magnetic flux through the ring. It is known that compared with the Andreev current, the Andreev conductance is more suitable to describe the Andreev reflection properties. Thus, we would like to investigate the Andreev conductance instead of the Andreev current. In the limit of zero temperature, the conductance expression can be written out, i.e.,

$$\mathcal{G} = \frac{\partial J}{\partial V} = \frac{2e^2}{h} T_{eh}(\omega = eV). \tag{8}$$

With the help of Eq. (5), we can work out the Green function responsible for the conductance, and then we can evaluate the Andreev-reflection properties in this structure.