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The effects of degeneracy of the carrier ensemble on the energy loss rate and the high field mobility characteristics under the conditions of low lattice temperatures

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ABSTRACT

The rate of loss of energy of the non-equilibrium electrons to the acoustic mode lattice vibration in a degenerate semiconductor is obtained under the condition, when the lattice temperature is low enough, so that the traditional approximations like the elastic nature of the electron-phonon collisions and the truncation of the phonon distribution to the equipartition law are not valid any more. Using the results of the energy loss rate, the non-ohmic mobility is then calculated. Evaluating the loss rate and the non-ohmic mobility in degenerate samples of Si and Ge we find that significant changes in both the characteristics have been effected compared to that in the non-degenerate samples, in the regime of lower energy and for relatively lower fields. The effected changes are more significant the lower the lattice temperature is.

1. Introduction

The electrical transport characteristic of a material decides on its suitability for a particular device. Under any prevalent conditions, the characteristics are determined by the dominant interactions of the electrons with the lattice imperfections. When the lattice temperature T_L is low ($T_L \leq 20$ K), the free electrons in a high purity elemental semiconductor interact dominantly only with intravalley acoustic phonons. Under this Condition, the electrons may be Significantly perturbed from the state of thermodynamic equilibrium even for a field of only a few V/cm or even less [1,2]. The non-equilibrium electrons then attain an effective temperature T_L which exceeds the lattice temperature and the material exhibits electrical non-linearity. The electrons then emit more phonons per unit time, compared to how much they absorb in the same interval. This leads to a finite rate of phonon growth, which results in a finite energy loss rate (ELR) of the ensemble of electrons.

In calculating the Phonon growth and the energy loss rate characteristics under the condition when the non-equilibrium electrons interact only with intravalley acoustic phonons, one traditionally neglects the Phonon energy ε_{ph} compared to the carrier energy $\varepsilon_{\vec{k}}$, i.e. assumes the electron-phonon interactions to be elastic and also approximates the phonon distribution by the equipartition law. For this long wave length acoustic phonon, it may be seen that $\varepsilon_{ph}/\varepsilon_{\vec{k}} \approx u_l/u_T$, where u_l is the acoustic velocity and the u_T is the average thermal velocity of the carriers [1]. Hence, though the traditional simplifications can be made at higher temperature the same simplifications can hardly be made if the temperature is low, where $u_T \approx u_l$. Hence under the condition of low temperature, the electron- phonon interaction can neither be assumed to be elastic, nor the phonon distribution be truncated to the equipartition form. It has been shown in [3,4] how do the approximations like the elastic interaction and the equipartition law for the phonon distribution lead to significant errors in the phonon growth and the energy loss characteristics in a non-degenerate (non-deg) semiconductor at low lattice temperatures.

For the low lattice temperatures if the Fermi energy ε_F is not much lower than the $k_B T_L$ of the conduction band edge (k_B being the Boltzmann constant), and the electron densities are beyond the insulator to metal transitions the free electrons ensemble in the semiconductor should be treated as degenerate (deg). With the increase of the doping level, as the electron concentration of an n-type material exceeds the effective density of states the Fermi level ε_F then moves into the conduction band and the material behaves as a degenerate one. The critical concentration of the donors N_D which is required for the degeneracy, may be roughly estimated from

$$\varepsilon_F = \left(\frac{\hbar^2}{2m^*}\right)(3\pi^2 N_D)^{2/3} > E_d$$

where m^* is the effective mass of an electron and E_d is the donor ionisation energy [5–7]. It may be kept in mind, though with the

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increase doping the interaction with the impurity atoms may be important, but such interaction being elastic, hardly takes part in the energy balance equation.

The purpose of the present communication is to calculate the energy loss rate characteristic, and then, from the loss-rate, to get the non-ohmic mobility characteristics in a degenerate sample of semiconductor at the low temperatures. The calculations have been carries out taking due account of the inelasticity of the electron-phonon interaction and also the true phonon distribution, and not truncating the same to the equipartition law. The numerical results which are obtained from the present theory for some degenerate samples of Si and Ge, are then compared with the results reported earlier for the non- degenerate materials in the same framework satisfying the low temperature conditions [3]. From the Comparison, the effects of degeneracy on the ELR and non-ohmic mobility characteristics are analyzed.

2. Development

The average rate of energy loss of a carrier due to interaction with the intravalley acoustic phonons can be calculated from [1]

$$\left\langle \frac{d\varepsilon_{\vec{k}}}{dt} \right\rangle = -\frac{1}{nV}\sum_{\vec{q}} \hbar u_l q \left(\frac{\partial N_q}{\partial t}\right) \tag{1}$$

where n is the concentration of the free carriers. V is the volume of the semiconductor material, $\hbar = \frac{h}{2\pi}$, h being the Plank's Constant, $\vec{q}$ is the phonon wave vector, N_q is the number of phonons with wave vector $\vec{q}$ and $(\frac{\partial N_q}{\partial t})$is the phonon growth rate. Now transforming the summation over $\vec{q}$ to an integration over the spherical coordinates q,θ, φ and integrating over, θ and φ one obtains

$$\left\langle \frac{d\varepsilon_{\vec{k}}}{dt} \right\rangle = -\frac{\hbar u_l}{2\pi^2 n}\int_{q=0}^{q_0} q^3 \left(\frac{\partial N_q}{\partial t}\right) dq \tag{2}$$

where q_0 is the upper limit of q. So, in order to carry out the integration in (2), apart from assigning a proper value of q_0, the expression for the phonon growth rate should be obtained for a degenerate semiconductor under the identical conditions of low temperature of our interest, where the effects of the inelasticity of the electron-phonon interaction and the true phonon distribution have been duly incorporated. One of the present authors, with some others have recently obtained such an expression for the phonon growth rate [8]. Making use of the expression for $(\frac{\partial N_q}{\partial t})$ from [8] one can obtain

$$\left\langle \frac{d\varepsilon_{\vec{k}}}{dt} \right\rangle = -\frac{(E_l m^*)^2 k_B{}^5 T_L{}^4 T_e}{4\pi^3 n \rho \hbar^7 u_l{}^4}(I_1 - I_2 - I_3) \tag{3}$$

where E_l is the deformation potential constant, ρ is the density and n, the electron concentration for the degenerate material is given by

$$n = \frac{2(2\pi m^* k_B T_e)^{3/2}}{8\pi^3\hbar^3} F_{\frac{1}{2}}(\eta_e)$$

$F_{\frac{1}{2}}(\eta_e)$ being the Fermi integral, $\eta_e = \frac{\varepsilon_F}{k_B T_e}$ and

$$I_1 = \int_0^{x_c} x^3 (N_q + 1) \quad \ln\left[1 + \exp\left\{\eta_e - a(x-b)^2 - \frac{x}{T_n}\right\}\right] dx \tag{4}$$

$$I_2 = \int_0^{x_c} x^3 N_q \quad \ln[1 + \exp\{\eta_e - a(x-b)^2\}] dx \tag{5}$$

$$I_2 = \int_0^{x_c} x^3 \left[\frac{1}{\lambda} \quad + \ln\lambda - 1 + \frac{x}{T_n}\left(\frac{1}{\lambda} - \frac{1}{2\lambda^2} - \frac{1}{2}\right)\right] dx \tag{6}$$

x being the normalised phonon wave vector given by $x = \frac{\hbar u_l q}{k_B T_L}$, $N_q = (e^x - 1)^{-1}$, $a = \frac{k_B T_L}{8 m^* u_l{}^2 T_n}$, $b = \frac{2 m^* u_l{}^2}{k_B T_L}$, $T_n = \frac{T_e}{T_L}$, $\lambda = 1 + \exp[\eta_e - a(x-b)^2]$.

The integrals (4)–(6) can be carried out analytically under the condition $\eta_e > \left[a(x-b)^2 + \frac{x_c}{T_n}\right]$

where x_c, the upper limit of the normalised phonon wave vector can be set at

$$x_c = \frac{2m^* u_l}{k_B T_L}\left[\sqrt{\frac{2k_B T_e \eta_e}{m^*}} - u_l\right]$$

Thus one can obtain

$$I_1 = (\eta - ab^2)\left[\frac{x_c{}^4}{4} - \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{3} P(3, r)\frac{x_c{}^{(3-r)}}{m^{(r+1)}} e^{-mx_c} - \frac{6}{m^4}\right\}\right] - \left(2ab - \frac{1}{T_n}\right) \left[\frac{x_c{}^5}{5} - \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{4} P(4, r)\frac{x_c{}^{(4-r)}}{m^{(r+1)}} e^{-mx_c} - \frac{24}{m^5}\right\}\right] - a\left[\frac{x_c{}^6}{6} - \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{5} P(5, r)\frac{x_c{}^{(5-r)}}{m^{(r+1)}} e^{-mx_c} - \frac{120}{m^6}\right\}\right]$$

$$I_2 = (ab^2 - \eta)\left[\sum_{r=0}^{3} P(3, r) x_c{}^{(3-r)} e^{-x_c} - 6 + \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{3} P(3, r)\frac{x_c{}^{(3-r)}}{(m+1)^{(r+1)}} e^{-(m+1)x_c} - \frac{6}{(m+1)^4}\right\}\right] - 2ab\left[\sum_{r=0}^{4} P(4, r) x_c{}^{(4-r)} e^{-x_c} - 24 + \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{4} P(4, r)\frac{x_c{}^{(4-r)}}{(m+1)^{(r+1)}} e^{-(m+1)x_c} - \frac{24}{(m+1)^5}\right\}\right] + a\left[\sum_{r=0}^{5} P(5, r) x_c{}^{(5-r)} e^{-x_c} - 120 + \sum_{m=1}^{\infty}\left\{\sum_{r=0}^{5} P(3, r)\frac{x_c{}^{(5-r)}}{(m+1)^{(r+1)}} e^{-(m+1)x_c} - \frac{120}{(m+1)^4}\right\}\right]$$

$$I_3 = e^{-\eta}\sum_{m=0}^{\infty}\frac{a^m}{m!}\left[\frac{1}{m_4}\{x_c'^{m_4} - (-b)^{m_4}\} + \frac{3b}{m_3}\{x_c'^{m_3} - (-b)^{m_3}\} + \frac{3b^2}{m_2}\{x_c'^{m_2} - (-b)^{m_2}\} + \frac{b^3}{m_1}\{x_c'^{m_1} - (-b)^{m_1}\}\right] + \frac{e^{-\eta}}{T_n}\sum_{m=0}^{\infty}(1 - 2^m e^{-\eta})\frac{a^m}{m!} H + \frac{1}{4}(\eta - ab^2 - 1)x_c{}^4 + \frac{2}{5}\left(ab - \frac{1}{4T_n}\right)x_c{}^5 - \frac{a}{6}x_c{}^6$$

where, $P(n,k) = \frac{n!}{(n-k)!}$

$m_i = 2m + 1$; i is a positive integer which ranges from 1 to 5

$x_c' = x_c - b$

$$H = \frac{1}{m_5}\{x_c'^{m_5} - (-b)^{m_5}\} + \frac{4b}{m_4}\{x_c'^{m_4} - (-b)^{m_4}\} + \frac{6b^2}{m_3}\{x_c'^{m_3} - (-b)^{m_3}\} + \frac{4b^3}{m_2}\{x_c'^{m_2} - (-b)^{m_2}\} + \frac{b^4}{m_1}\{x_c'^{m_1} - (-b)^{m_1}\}$$

Now $\left\langle \frac{d\varepsilon_{\vec{k}}}{dt} \right\rangle_{deg}$, the average energy loss rate of the electrons in the degenerate material due to the phonon emission being known, one can obtain the non-ohmic mobility μ.

In the presence of an electric field E, the energy supplied to the carriers is at the rate $e\mu E^2$. A steady state is reached when the average energy loss rate due to phonon emission, balances the rate of gain of energy from the field [1]

$$\left\langle \frac{d\varepsilon_{\vec{k}}}{dt} \right\rangle = e\mu E^2 \tag{7}$$

Thus the non-ohmic mobility of the degenerate semiconductors under the prevalent conditions of low temperature when the inelastic interaction of the electrons with the intravalley acoustic phonons and the full form of the Bose-Einstein distribution for the phonons are duly