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A theoretical study on critical phenomena of magnetic soft modes

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ABSTRACT

Below a threshold magnetic field, domain structures in ferromagnetic samples may start to nucleate from the initially saturated state via either continuous or discontinuous phase transitions. Such processes are usually accompanied by the occurrence of soft spin-wave modes at the critical point. In this paper, we present a theoretical study on the critical phenomena of uniform soft modes in a macrospin model and spatially non-uniform ones in ferromagnetic thin films. The critical exponents of the mode frequency and its polarization are derived. The value is found to be equal to one half, which is directly related to the breaking of a reflection-symmetry in the phase transition. At the critical point, the soft mode becomes linearly polarized, which provides an additional measurable effect of the critical phenomena.

1. Introduction

Depending on size, shape, material or external field, ferromagnets can form a wide variety of spin configurations, which are usually called magnetic domains. Field evolution of the domain structures has been an important topic in magnetism, due to both fundamental interests and technical relevance. For instance, the magnetization reversal of magnetic particles involves subsequent formation of stable or metastable spin configurations. As pointed out in Ref. [1], "a convenient way to describe their magnetization process is to gather the possible configurations into micromagnetic phases and determine the related phase transitions."

In the past two decades, a great number of studies on magnetization dynamics in confined magnetic structures have been carried out. Many of them focused on the collective excitations of the spin system at various equilibrium states under different external fields. One particularly interesting observation is the softening of certain spin-wave modes, which occurs when the system undergoes a transition between different magnetic structures [1–10]. Examples include domain nucleation from saturation in magnetic thin-film strips [1–5], magnetization reversal of elliptical nanoparticles [6,7], and vortex formation and annihilation in various nanostructures [8–14]. All those transitions, which could be either continuous or discontinuous, were found to be triggered by soft-mode instability. Therefore, the occurrence of soft modes can be considered as a direct indication of the phase transitions. In this paper, we analytically study the critical phenomena of the soft modes, in both a macrospin model and a discretized model for ferromagnetic thin films. The critical exponent of the soft mode is derived to be equal to one half in the case that a reflection-symmetry is broken in the phase transition. Previous studies focused on the vanishing of the mode frequency at the critical point. Here we point out another tightly-related effect, namely, the polarization of the mode profile (definition given below). In a phase transition that breaks a reflection symmetry, the associated soft mode becomes linearly polarized with the critical exponent equal to one half as well. The linear polarization of the soft mode, resulting in the vanishing of one dynamic component of the magnetization, thus provides a measurable effect equivalent to the vanishing of the frequency.

The paper is structured in the following manner. The first section provides a brief introduction of our theoretical approach for solving the normal modes of a magnetic sample. The second section deals with a simple case, i.e., the softening of the uniform mode in a macrospin model. Here we introduce the idea of the mode polarization. In the third section, we discuss the critical phenomena of the spatially non-uniform soft modes in ferromagnetic thin films. Finally we summarize our results.

2. Theoretical method

In our paper, the critical phenomena of the soft modes are discussed in the framework of the so-called dynamical matrix approach [15,16], which allows the solution of the normal modes of a magnetic sample at any given ground state. This method is a discretized form of solving the linearized equation of motion of the magnetization based on

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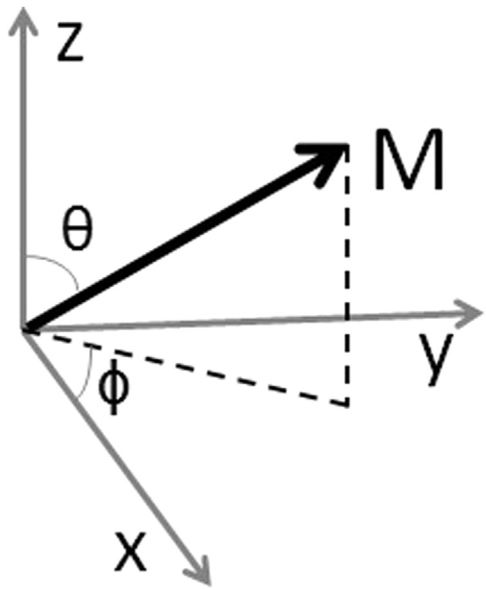

Fig. 1. Polar angles in a Cartesian coordinate system..

the energy formulation. In this approach, the magnetization vector $\vec{m}$ and the free energy density E of the system are expressed in terms of its polar angles φ and θas defined in Fig. 1. In the simplest case, the magnetic sample under consideration can be treated as a single dipole, or a macrospin. The only normal mode of the system is the uniform mode, usually referred as the ferromagnetic resonance (FMR) mode, which is the precessional motion of the magnetization around its equilibrium orientation ... The deviations from the equilibrium are indicated by small angles $\delta\varphi$ and $\delta\theta$. Then, the eigen frequency (Ω) and the eigenvector $(\delta\varphi, \delta\theta)^T$of the resonance mode can be obtained from the solution of two linear equations [17]:

$$\begin{cases} \frac{i\Omega M_s}{\gamma}\sin\theta_0\delta\theta = E_{\varphi\varphi}\delta\varphi + E_{\varphi\theta}\delta\theta \\ \frac{-i\Omega M_s}{\gamma}\sin\theta_0\delta\varphi = E_{\varphi\theta}\delta\varphi + E_{\theta\theta}\delta\theta, \end{cases} \tag{1}$$

where$E_{\alpha\beta}$ are the second derivatives of the total energy with respect to the polar angles at the equilibrium state, $M_s$is the saturation magnetization, and γ the gyromagnetic ratio. In the general case, one has to solve all the normal modes of a magnetic sample, including also spatially non-uniform ones. This can be approached by discretizing the magnetic sample into N equal size, rectangular parallelepipeds cells. In a chosen reference frame, a particular configuration of the sample can be described by a vector $\Phi = \{\varphi_n, \theta_n\} = (\varphi_1, \theta_1, \cdots, \varphi_N, \theta_N)^T$, where the magnetization in the nth cell is given by

$$\vec{m}_n = (m_n^x, m_n^y, m_n^z) = (\sin\theta_n\cos\varphi_n,\ \sin\theta_n\sin\varphi_n,\ \cos\theta_n). \tag{2}$$

At the ground state defined by Φ, the normal modes of the discretized system can be calculated by solving an eigenvalue problem [15],

$$Bv = \lambda v, \tag{3}$$

where $\lambda = iM_s\Omega/\gamma$ and $v = (\delta\varphi_1, \delta\theta_1, \cdots, \delta\varphi_N, \delta\theta_N)^T$ is a vector consisting of the small deviations of the polar angles of each cell from its equilibrium direction. The elements of matrix B are given by

$$B_{2k,1,2j-1} = -\frac{E_{\theta k\phi j}}{\sin\theta_k},\ B_{2k-1,2j} = -\frac{E_{\theta k\theta j}}{\sin\theta_k},$$
$$B_{2k,2j-1} - 1 = \frac{E_{\phi k\phi j}}{\sin\theta_k},\ B_{2k,2j} = \frac{E_{\phi k\theta j}}{\sin\theta_k}, \tag{4}$$

for k, j=1, ⋯, N[9], which must be evaluated at the equilibrium state Φ. The solutions of Eq. (3) yield both the frequency of each mode and its corresponding profile, given by the eigenvector v. In the general case, v is complex since it contains information on both the amplitude and relative phase of the precession in each cell, from which the dynamical precession of each mode can be constructed. Solving Eq. (3) for normal modes usually requires numerical efforts.

The dynamical matrix approach explicitly associates the normal modes with the energy formulation of the system. On the other hand, it is well known that phase transitions are closely related to the energy behavior near the critical point. Therefore, the dynamical matrix approach provides a suitable method to analyze the critical phenomena of the soft modes that induce micromagnetic phase transitions. In the following section, we first study a macrospin system, which has a uniform soft mode. This model allows a simple discussion of the critical phenomena of the soft mode and its connection with the related phase transition.

3. Soft modes in a macrospin model

We consider a spherical crystal magnetized by a static magnetic field and focus on the normal excitations of the system during its field evolution. Suppose the sphere has an uniaxial anisotropy with the easy direction along the z axis. A static field is applied along the x axis. In a macrospin model, the sample is treated as a single magnetic dipole, which is characterized by the direction of its magnetization. Depending on the external field, the magnetization of the sphere points to different directions to minimize the total energy of the system, defining the corresponding equilibrium state. At a given equilibrium, the system has only one normal mode, i.e., the FMR mode.

The calculation of the mode frequency of this system has been presented in Ref. [17] as an example of the energy formulation of the equations of motion. Here we briefly repeat the calculations and make further analysis. Following Ref. [17], the energy density of the sphere can be written in terms of polar angles as

$$E = K_1'\sin^2\theta - HM_s\sin\theta\cos\varphi, \tag{5}$$

whereK_1' is a positive anisotropy constant and H is the static field.

The field evolution of the system can be obtained by minimizing the energy with respect to φ and θ, which yields the equilibrium orientation at different external fields [17]

$$\begin{cases} \varphi = 0,\ \sin\theta = \frac{HM_s}{2K_1'}, & for\quad H < \frac{2K_1'}{M_s}; \\ \varphi = 0,\ \theta = \frac{\pi}{2}, & for\quad H \geq \frac{2K_1'}{M_s}. \end{cases} \tag{6}$$

It is clear that the system goes through a continuous phase transition at a critical field $H_c \equiv \frac{2K_1'}{M_s}$. Above H_c, the sample is saturated along the field direction ($\theta = \frac{\pi}{2}$). Just below H_c, the magnetization starts to deviate from the field direction by an infinitesimal angle δ, which satisfies $\sin(\frac{\pi}{2} + \delta) = \frac{H}{H_c}$. By a Taylor expansion, one sees that $\delta \propto \sqrt{H_c - H}$. One can also make a Taylor expansion of the energy near the critical point in terms of $\delta$from $\theta = \frac{\pi}{2}$, which reads

$$E|_{\theta=\frac{\pi}{2}+\delta,\varphi=0} \approx (K_1' - HM_s) + \frac{M_s}{2}(H - H_c)\delta^2 + \frac{M_s}{4!}(4H_c - H)\delta^4. \tag{7}$$

On the right-hand side, the cubic term of δ vanishes due to the symmetry of the system. It is obvious that if $\delta$is defined as the order parameter, then Eq. (7) resembles the energy formulation near the critical point in Landau's second order phase transition theory [18,19].

We now turn to the mode frequency of the system, which can be calculated by requiring the vanishing of the determinant of the coefficients of $\delta\varphi$ and $\delta\theta$ in Eq. (1). The results are given in Ref. [17] as

$$\begin{cases} \Omega^2 = \gamma^2(H_c^2 - H^2), & for\quad H < H_c; \\ \Omega^2 = \gamma^2H(H - H_c), & for\quad H \geq H_c. \end{cases} \tag{8}$$

This yields the softening of the resonance mode at H_c and the critical exponent of the mode frequency, which is equal to one half.

Besides the order parameter and the mode frequency as discussed