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Quantum-critical scaling of fidelity in 2D pairing models

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ABSTRACT

The laws of quantum-critical scaling theory of quantum fidelity, dependent on the underlying system dimensionality D, have so far been verified in exactly solvable $1D$ models, belonging to or equivalent to interacting, quadratic (quasifree), spinless or spinfull, lattice-fermion models. The obtained results are so appealing that in quest for correlation lengths and associated universal critical indices ν, which characterize the divergence of correlation lengths on approaching critical points, one might be inclined to substitute the hard task of determining an asymptotic behavior at large distances of a two-point correlation function by an easier one, of determining the quantum-critical scaling of the quantum fidelity. However, the role of system's dimensionality has been left as an open problem. Our aim in this paper is to fill up this gap, at least partially, by verifying the laws of quantum-critical scaling theory of quantum fidelity in a $2D$ case. To this end, we study correlation functions and quantum fidelity of $2D$ exactly solvable models, which are interacting, quasifree, spinfull, lattice-fermion models. The considered $2D$ models exhibit new, as compared with $1D$ ones, features: at a given quantum-critical point there exists a multitude of correlation lengths and multiple universal critical indices ν, since these quantities depend on spatial directions, moreover, the indices ν may assume larger values. These facts follow from the obtained by us analytical asymptotic formulae for two-point correlation functions. In such new circumstances we discuss the behavior of quantum fidelity from the perspective of quantum-critical scaling theory. In particular, we are interested in finding out to what extent the quantum fidelity approach may be an alternative to the correlation-function approach in studies of quantum-critical points beyond 1D.

1. Introduction

In recent years, quantum phase transitions and quantum-critical phenomena constitute a subject of great interest and vigorous studies in condensed matter physics. Both, experimental and theoretical developments point out to the crucial role that quantum phase transitions play in physics of frequently studied high-T_c superconductors, rare-earth magnetic systems, heavy-fermion systems or two-dimensional electrons liquids exhibiting fractional quantum Hall effect [1,2]. Quantum-critical phenomena have been also observed in exotic systems as magnetic quasicrystals [3] and in artificial systems of ultracold atoms in optical lattices [4]. The so called classical, thermal phase transitions originate from thermal fluctuations, a competition of internal energy and entropy, and are mathematically manifested as singularities in temperature and other thermodynamic parameters of various thermodynamic functions, and such characteristics of correlation functions as the correlation length, at nonzero temperatures. In contrast, quantum phase transitions originate from purely quantum fluctuations and are mathematically manifested as singularities in system parameters of the ground-state energy density, which is also the zero-temperature limit of the internal energy density. Naturally, singularities of thermodynamic functions appear only in the thermodynamic limit. The importance of quantum phase transitions for physics and the related wide interest in such transitions stems from the fact that, while a quantum phase transition is exhibited by ground states, hence often termed a zero-temperature phenomenon, its existence in a system exerts a great impact on the behavior of that system also at nonzero temperatures. A quantum-critical point gives rise to the so called quantum-critical region, which extends at nonzero temperatures, in some cases up to unexpectedly high temperatures [5,6].

Theoretically, quantum phase transitions can be studied in quite complex quantum systems by qualitative and approximate methods, or in relatively simple but exactly solvable models by means of analytic methods and high-accuracy numerical calculations [1]. Naturally, for the purpose of testing and illustrating general or new ideas the second route is most suitable. Traditionally, this route involves studying the eigenvalue problem of a Hamiltonian, the ground state and excitation

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gaps, determining quantum-critical points and symmetries, constructing local-order parameters, calculating two-point correlation functions and their asymptotic behavior at large distances and in vicinities of quantum-critical points, with correlation lengths and the universal critical indices ν that characterize the divergence of correlation lengths on approaching critical points. Carrying out such a program is a hard task, which has been accomplished only in a few one-dimensional models. Among those models, there are quantum spin chains as the isotropic and anisotropic XY models in an external transverse magnetic field, including their extremely anisotropic version–the Ising model [1]. Only in one dimension those models are equivalent to lattice gases of spinless fermions, which can exactly be diagonalized, and exact results concerning the phase diagram, quantum-critical points, correlation functions and dynamics have been obtained (concerning XY model see [7–9], concerning the Ising model see [10], and for both models [11]). Needless to say that parallel results for a higher-dimensional model are desirable; this is the first motivation of our investigations presented in this paper.

In the last decade, fresh ideas coming from quantum-information science entered the field of quantum phase transitions. One of them is the so called quantum-fidelity method. While in this paper we are considering known, well controlled continuous quantum phase transitions – quantum critical points, it is worth to note that the fidelity method has been successfully used to detect various quantum phase transitions in numerous models. In contrast to standard methods no a priori knowledge of symmetries, order parameters, etc. is required, as is the case for instance in quantum topological phase transitions [28]. The carried out so far studies of particular models show that using this method it is possible to not only locate critical points [12–15] but also to determine the correlation lengths and associated universal critical indices ν [16–18]. This is achieved by studying (typically by numerical methods) quantum-critical scaling properties, with respect to the size of the system and the parameters of the underlying Hamiltonian, of the quantum fidelity of two ground states in a vicinity of a quantum-critical point. The index ν is extracted from numerically obtained plots of fidelity via the quantum-critical scaling laws of quantum fidelity, which have been derived by renormalization group arguments [16,19,21,22]. The most comprehensive results concerning the fidelity approach have been obtained for one-dimensional quantum spin systems in a perpendicular magnetic field [16] (the case of Ising model), [17] (the case of XY model), [18] (the case of a quasifree, pairing, lattice-fermion model). These results are very promising: except a vicinity of a multicritical point, the fidelity approach works fine. Since the task of determining quantum-critical scaling properties of the quantum fidelity is definitely much easier than the task of calculating large-distance asymptotic behavior of two-point correlation functions, one is tempted to consider the fidelity method as a substitute for the standard correlation-function approach. All that we have said above makes it desirable to verify the laws of quantum-critical scaling of fidelity and the effectiveness of fidelity approach in dimensions higher than one, where new features, not encountered in one-dimensional models may appear; this is the second motivation of our investigations reported in this paper. For that task we need an at least two-dimensional exactly solvable model, whose ground states, quantum-critical points with critical indices in their vicinities, correlation lengths, and analytic expressions for fidelity can be determined.

To go beyond the one-dimensional case, we consider lattice-fermion models which originate from the two-dimensional model of d-wave superconductivity proposed by Sachdev [23](see also [1]), which are spinful pairing models represented by quadratic Hamiltonians. In many cases of physical interest the quadratic Hamiltonians are obtained by means of a mean-field approximation applied to quartic Hamiltonians of systems with two-body interactions and their parameters are related by self-consistency equations. While our Hamiltonians are also quadratic, their parameters are independent—not related by mean-field equations. General, mathematical considerations of some classes of such models, but without specifying hopping intensities or coupling constants, which therefore do not reach such subtleties as quantum-critical points or critical behavior of correlation functions, can be found in [24,25]. For translation-invariant hopping intensities and coupling constants the considered models are exactly solvable in any dimension, that is, in particular it is possible to derive analytical formulae for quantum fidelity and correlation functions of finite systems and then in the thermodynamic limit, where boundary conditions play no role. To limit further the great variety of possible models, we restrict the hopping intensities to nearest neighbors while the dimensionality is set to D=2. The underlying lattice is chosen as a square one with hopping intensities invariant under rotations by $\pi/2$, while the interactions of our systems are required not to extend beyond nearest neighbors and to be either invariant under rotations by $\pi/2$ (the symmetric model) or to change sign after such a rotation (the antisymmetric model).

It is worth to mention here another class of models, which might be of interest in the context of this paper, known as reduced BCS models of superconductivity and superfluidity, see for instance [26] and references quoted there. While those models are exactly integrable, analytical formulae for quantities of interest, such as correlation functions, are not available, except at the thermodynamic limit, where they coincide with the results of mean-field theories. For finite systems, all the quantities we are interested in, ground-states, quantum fidelity, ground-state correlation functions, are given in terms of only numerically available solutions of a set of coupled, nonlinear, algebraic equations, whose number amounts to the number of degrees of freedom. Therefore, despite some interesting features, for instance the ground state is not a BCS-like state, those models are not suitable for the kind of studies reported in this paper.

The general plan of the paper is as follows. In Section 2 we define the two models studied in this paper, the symmetric model and the antisymmetric one, and give closed-form formulae for two basic ground-state two-point correlation functions, which are then used to define order parameters of those models. Later on, when specific asymptotic behavior of two-point functions is discussed, only the gauge invariant function, that is the offdiagonal matrix element of the ground-state one-body reduced density operator is taken into account. The purpose of the next Section 3 is to present the quantum fidelity method of investigating quantum-critical points. In particular we provide the known quantum-critical scaling laws obeyed by fidelity in a vicinity of a critical point, and express the fidelity of the models considered in the paper as a Riemann sum of an analytically given function of two variables – the components of quasimomentum. Then, in Sections 4 and 5 we carry out our program of confronting predictions of quantum-critical scaling theory of fidelity with exact results obtained for 2D pairing models, the symmetric and antisymmetric ones. This program consists of two stages. Since a critical scaling theory is concerned with correlation lengths and critical indices, in the first stage analytic results for the spatial direction-dependent behavior of the gauge-invariant two-point correlation function at sufficiently large spatial distances and sufficiently close to the quantum-critical points exhibited by our models are highly desirable. Having such results, one is able to directly infer exact expressions, in terms of system's parameters, for spatial direction-dependent correlation lengths and exact values of the corresponding critical indices. We provide such results in the paper; they are excerpts from our article [32], where comprehensive studies of the gauge-invariant two-point correlation function of our models have been carried out. In the second stage, we are concerned with the variations of quantum fidelity against the system's linear size or against the distance to a critical point, sufficiently close to a quantum-critical point. Having expressed the fidelity as a Riemann sum, we generate suitable high-accuracy numerical plots that reveal those variations. Then, our discussion concentrates on answering two questions. First, can we read off from those plots the values of the spatial direction-dependent correlation lengths?