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The dynamics of magnetic vortices in type II superconductors with pinning sites studied by the time dependent Ginzburg–Landau model

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ABSTRACT

We investigate the dynamics of magnetic vortices in type II superconductors with normal state pinning sites using the Ginzburg–Landau equations. Simulation results demonstrate hopping of vortices between pinning sites, influenced by external magnetic fields and external currents. The system is highly nonlinear and the vortices show complex nonlinear dynamical behaviour.

1. Introduction

The dynamics of magnetic vortices in type II superconductors at temperatures close to the critical temperature can be modelled by the time dependent Ginzburg–Landau equations. The theory is based on a Schrödinger type equation with a potential containing a quadratic term and a quartic term in addition to a kinetic term involving the momentum operator coupled to a magnetic field governed by the Maxwell equations [1–3]. For type-II superconductors the Ginzburg–Landau equations model the magnetic field penetration through quantized current vortices as the externally applied magnetic field exceeds a threshold value. A number of variants of the Ginzburg–Landau equations have been used to investigate pattern formation in different nonlinear media, not only in superconductivity, and hence have become a popular field of study in nonlinear science [4,5]. Our aim here is to investigate the dynamics of vortices in the presence of normal state pinning sites in the superconductor [3,6]. Such pinning sites can arise from atomic impurities, magnetic impurities, lattice defects and defects in general. The Gibbs energy of the superconductor is given by a 4th order potential in the order parameter. The sign of the coefficient to 2nd order term determines the phase transition between the normal and the superconducting state and hence this coefficient can be used to fix the positions of inserted pinning sites. Secondly, we shall present a model for the action of the self induced magnetic field on vortex generation, when a net current is flowing through a superconductor enforced by metal leads at the ends of a superconducting strip.

2. The time dependent Ginzburg–Landau model

The superconducting state is described by the order parameter $\psi(\mathbf{r}, t)$, where $\mathbf{r}$ is the position in the superconducting volume denoted $\Omega \subset R^3$ and t is time. In the framework of the Ginzburg–Landau theory the Gibbs energy of the superconducting state G_s is given by

$$G_s = G_n - \alpha_0(\mathbf{r})\left(1 - \frac{T}{T_c}\right)|\psi|^2 + \frac{\beta}{2}|\psi|^4. \quad (1)$$

Here G_n is the Gibbs energy of the normal state, T is the absolute temperature and T_c is the critical temperature. The parameter β is a constant and $\alpha_0(\mathbf{r})$ we choose such that it depends on the space variable $\mathbf{r}$ in order to model pinning sites depleting the superconducting state at specific positions. For $T < T_C$ positive values of α_0 correspond to the superconducting state and negative values model a pinning site at which the superconducting state becomes normal.

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The order parameter is influenced by the magnetic field $\mathbf{B}(\mathbf{r}, t)$ given by the magnetic potential through the relation $\mathbf{B} = \nabla \times \mathbf{A}$. The Ginzburg–Landau parameter κ is introduced as the ratio between the magnetic field penetration length λ and the coherence length ξ, i.e. $\kappa = \lambda/\xi$. We shall investigate the dynamics of flux vortices penetrating the superconductor in the presence of pinning sites, whose positions are given by a function $f(\mathbf{r})$ taking the value one in the superconducting regions and the value -1 at the position of a pinning site. After scaling to normalized coordinates and using the zero electric potential gauge the Ginzburg–Landau equations for the order parameter in nondimensional form reads [7]

$$\frac{\partial \psi}{\partial t} = -\left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)^2 \psi + f(\mathbf{r})\psi - |\psi|^2\psi, \tag{2}$$

$$\sigma\frac{\partial \mathbf{A}}{\partial t} = \frac{1}{2i\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) - |\psi|^2\mathbf{A} - \nabla\times\nabla\times\mathbf{A}. \tag{3}$$

In order to make the Ginzburg–Landau equations dimensionless, we have scaled the space coordinates by the magnetic field penetration depth λ and time is scaled by ξ^2/D, where D is a diffusion coefficient [8]. The magnetic field $\mathbf{A}$ is scaled by the factor $\hbar/(2e\xi)$, where e is the electron charge. The wave function ψ is scaled by $\sqrt{\alpha_0/\beta}$. The term in σ is the conductivity of the normal current of unpaired electrons. It is scaled by the factor $1/(\mu_0 D\kappa^2)$, where μ_0 is the magnetic permeability of the free space. The normal current $\mathbf{J}_n$ and the super current $\mathbf{J}_s$ read

$$\mathbf{J}_n = -\sigma\frac{\partial \mathbf{A}}{\partial t} \quad \text{and} \quad \mathbf{J}_s = \frac{1}{2i\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) - |\psi|^2\mathbf{A}, \tag{4}$$

with the total current being $\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s$. The coefficient $f(\mathbf{r})$ of ψ in Eq. (2) defines the positions of the pinning sites by changing sign from $+1$ to -1 and through scaling f is related to α_0 in Eq. (1). In solving numerically Eqs. (2) and (3) we need to specify appropriate boundary conditions. We seek to satisfy the following three boundary conditions on the boundary of the superconducting region $\partial\Omega$

$$\nabla\times\mathbf{A} = \mathbf{B_a}, \quad \nabla\psi\cdot\mathbf{n} = 0 \quad \text{and} \quad \mathbf{A}\cdot\mathbf{n} = 0. \tag{5}$$

The first condition tells that the magnetic field at the surface of the superconductor equals the applied external field $\mathbf{B}_a$. The condition $\nabla\psi\cdot\mathbf{n} = 0$ corresponds to no super current crossing the boundary. Differentiating the boundary condition $\mathbf{A}\cdot\mathbf{n} = 0$ with respect to time shows that this condition prevents normal conducting current to pass the boundary.

The Ginzburg–Landau Eqs. (2) and (3) have been solved using the finite element software package COMSOL Multiphysics [9,10]. In order to model pinning sites we have introduced the function $f(\mathbf{r})$ with real values between -1 and $+1$, where -1 corresponds to a normal state pinning site and $+1$ corresponds to regions of the superconducting state. We have chosen f to take the phenomenological form

$$f(\mathbf{r}) = \Pi_{k=1}^{N} f_k(\mathbf{r}) \quad \text{where} \quad f_k(\mathbf{r}) = \tanh((|\mathbf{r} - \mathbf{r}_{0k}| - R_k)/w_k). \tag{6}$$

The function f attains the values -1 around N pinning sites positioned at $\mathbf{r}_{0k}$, for $k = 1, 2, \ldots, N$. In the following we shall consider a two dimensional superconductor where $\mathbf{r} = (x, y)$ and $\mathbf{r}_{0k} = (x_{0k}, y_{0k})$. This means that the pinning sites are circular with radius R_k and the transition from the superconducting state to the normal state happens within an annulus of width w_k. Assuming a two dimensional superconductor means we strictly study an infinite prism, where the currents are flowing in parallel with the xy-plane and the magnetic field is perpendicular to the xy-plane. However, the approach is valid for sufficiently thick finite size superconductors, where the geometric edge effects are negligible. If the thickness is denoted by t then $t > > \lambda$.

Other choices for modelling pinning sites are available in the literature. In particular we mention the local reduction of the mean free electron path at pinning centres included in the Ginzburg–Landau model by Ge et al. [11]. Here the mean free path enters as a factor on the momentum term in Eq. (2). This approach is more based on first principles in the physical description than ours. The above modelling strategy may also be used to investigate suppression of the order parameter. In particular we mention the experimental work by Haag et al. [6], where regular arrays of point defects have been inserted into a superconductor by irradiation with He^+ ions. These defects acts as pinning sites.

Numerical simulations. In Fig. 1 we show snapshots of $|\psi|^2$ from one simulation of the time dependent Ginzburg–Landau Eqs. (2) and (3) subject to the boundary conditions (5) from time $t = 0$ until $t = 750$. We have chosen the initial conditions $\psi(\mathbf{r}, 0) = (1+i)/\sqrt{2}$ and $\mathbf{A} = (0, 0)$. The external magnetic field $\mathbf{B}_a$ is turned on at time $t = 0$. This leads to a discontinuous mismatch between the initial vanishing magnetic field within the superconductor and the external applied magnetic field. The algorithm can handle this without problems. Alternatively one could turn on the external magnetic field gradually giving a more smooth transition.

In the region of interest we have inserted 4 pinning sites denoted d1, d2, d3 and d4 at the respective positions $\mathbf{r}_{01} = (-1, -3)$, $\mathbf{r}_{02} = (0, -2)$, $\mathbf{r}_{03} = (1, -1)$ and $\mathbf{r}_{04} = (2, 0)$. The pinning sites are modelled by f in Eq. (6) using $R_k = 0.2$ and $w_k = 0.05$ for $k = 1, 2, 3, 4$. The external applied magnetic field $B_a = 0.73$ is chosen slightly smaller than the critical magnetic field for a superconductor with no pinning sites. This value leads to very complex dynamics of fluxons entering the superconductor resulting from mutual interactions and interactions with the pinning sites as illustrated in Fig. 1. At time $t = 12$ we observe a fluxon, f1, entering the superconductor, hopping from d1 to d2 influenced by repulsive forces from the boundary and attractive forces from the pinning sites. At time $t = 72$ a second fluxon, f2, has entered the superconductor and are attached to the pinning site d1 and eventually pushing the first fluxon f1 onto d3. As time progress the fluxon f2 deattaches d1 and moves into the superconductor and at the same time a third fluxon, f3, enters the superconductor at the right hand side moving toward d4, where it becomes trapped. A fourth fluxon, f4, enters at the bottom boundary close to d1 and propagates into the superconductor and away from the pinning sites. Finally, a fifth fluxon, f5, enters close to d1 and hops from d1 to d2, where it finally gets trapped. At $t = 750$ we have obtained a stationary state with two fluxons in the bulk superconductor and three fluxons trapped on the pinning sites d2, d3 and d4. No fluxon is attached to d1. In short the simulation results in Fig. 1 illustrate the intricate nonlinear dynamical behaviour of fluxons hopping from pinning site to pinning site and at the same time experience mutual repulsive forces and repulsive forces from the boundaries, controlled by the external applied magnetic field.

3. Current carrying superconducting strips

In this section we study superconducting strips carrying currents along the strip. The current is injected through metal contacts at two opposite boundaries of the superconductor, that is at $x = -L_x$ and $x = L_x$, respectively, where L_x is the length of the superconductor in the x-axis direction. At the side boundaries the superconducting current and the normal current are parallel to the superconductor surface and therefore we use here the boundary conditions [1,12].

$$\nabla\times\mathbf{A} = \mathbf{B_e} = \mathbf{B}_a + \mathbf{B}_c, \quad \nabla\psi\cdot\mathbf{n} = 0 \quad \text{and} \quad \mathbf{A}\cdot\mathbf{n} = 0. \tag{7}$$

In the above equations $\mathbf{B}_e$ is the total external magnetic field composed of the sum of the applied magnetic field $\mathbf{B}_a$ and the magnetic field $\mathbf{B}_c$ induced from the total current $\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n$ in the

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