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Flux-induced Nernst effect in low-dimensional superconductors

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ABSTRACT

A method is available that enables consistent study of the stochastic behavior of a system that obeys purely diffusive evolution equations. This method has been applied to a superconducting loop with nonuniform temperature, with average temperature close to T_c. It is found that a flux-dependent average potential difference arises along the loop, proportional to the temperature gradient and most pronounced in the direction perpendicular to this gradient. The largest voltages were obtained for fluxes close to $0.3\Phi_0$, average temperatures slightly below the critical temperature, thermal coherence length of the order of the perimeter of the ring, BCS coherence length that is not negligible in comparison to the thermal coherence length, and short inelastic scattering time. This effect is entirely due to thermal fluctuations. It differs essentially from the usual Nernst effect in bulk superconductors, that is induced by magnetic field rather than by magnetic flux. We also study the effect of confinement in a 2D mesoscopic film.

1. Introduction

In 1886, while studying the Hall effect in bismuth, Ettingshausen and Nernst [1] noticed an unexpected perpendicular current flow when one side of the sample was heated.

The Nernst effect in low temperature superconductors was detected long ago [2], and in high temperature superconductors the effect was detected [3] soon after their discovery. Ullah and Dorsey [4,5] described the effect theoretically by means of the time-dependent Ginzburg–Landau model (TDGL). Nernst signals have been observed in an extended region above the critical temperature in high-T_c materials [6] and in conventional superconductors [7].

We will see that thermal fluctuations are an essential ingredient for the appearance of the Nernst effect in the situations that we will study. The following section is therefore devoted to describe how these fluctuations can be taken into account.

2. Thermal fluctuations in quasi-1D superconducting samples

The simplest model for the description of the evolution of a superconducting sample is TDGL [8,9]. This description involves a complex field, dubbed 'the order parameter,' ψ, and the vector potential, $\mathbf{A}$. We choose a gauge in which the scalar electric potential vanishes. In a numerical analysis, space is discretized into small cells, and the order parameter and the vector potential in cell number j are denoted by ψ_j, and $\mathbf{A}_j$. TDGL asserts that the evolution of ψ_j is given by

$$\frac{d\,\mathrm{Re}\psi_j}{dt} = -\Gamma_{\psi,j}\frac{\partial G}{\partial \mathrm{Re}\psi_j}\ ; \quad \frac{d\,\mathrm{Im}\psi_j}{dt} = -\Gamma_{\psi,j}\frac{\partial G}{\partial \mathrm{Im}\psi_j}, \tag{1}$$

where t is the time, $\Gamma_{\psi,j}$ is the relaxation rate, and G is the free energy of the system. For a wire of length L, with a uniform cross section that is sufficiently small to neglect the screening of the magnetic field, divided into N cells of equal length, with periodic boundaries and appropriate normalization of the order parameter, the free energy can be written as

$$G = \frac{Lk_BT_c}{N\xi_\beta}\sum_{j=0}^{N-1}\left\{\frac{\xi_\beta^2(T_j-T_c)}{\xi^2(0)T_c}|\psi_j|^2 + \frac{1}{2}|\psi_j|^4 + \frac{N^2\xi_\beta^2}{L^2} \times \left((2+\tilde{A}_j^2)|\psi_j|^2 - \mathrm{Re}[\psi_j^*(2+\mathrm{i}\tilde{A}_j+\mathrm{i}\tilde{A}_{j+1})\psi_{j+1}]\right)\right\} + \frac{I\hbar}{2e}\sum_{j=0}^{N-1}\tilde{A}_j, \tag{2}$$

where k_B is the Boltzmann constant, T_j is the temperature at site j, $\xi(0)$ is the coherence length at zero temperature, $\tilde{A}$ is the component of $\mathbf{A}$ along the wire multiplied by $2eL/\hbar cN$, e is the absolute value of the electronic charge, c is the speed of light, the asterisk denotes complex conjugation, I is the total current along the sample, and the length ξ_β is

$$\xi_\beta = \left(\frac{a\Phi_0^2}{32\pi^3\kappa^2k_BT_c}\right)^{1/3}, \tag{3}$$

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where a is the cross sectional area of the sample, $\Phi_0 = \pi\hbar/ec$ is the quantum of flux, and κ is the Ginzburg–Landau parameter. For a uniform wire the relaxation rate is independent of j and its value is $\Gamma_\psi = ND/2\xi_\beta Lk_BT_c$, where D is the diffusivity.

The meaning of ξ_β is that of the typical 'length of a fluctuation.' With the notation of [10], and ignoring the electromagnetic field, the free energy of the superconducting wire at $T = T_c$ is given by

$$G = a\int \mathrm{d}s\left(\frac{\beta}{2}|\Delta|^4 + \gamma\hbar^2\left|\frac{\mathrm{d}\Delta}{\mathrm{d}s}\right|^2\right), \quad (4)$$

where s is the arc length along the wire and Δ is the order parameter. The first term in this expression "pushes" the order parameter towards zero, and the second term pushes it towards uniformity, but due to thermal fluctuations, there will be regions where Δ differs appreciably from zero. Denoting by $\bar{\Delta}$ the typical value of $|\Delta|$ in such regions and by ξ_β the typical extension of these regions, the typical value of $|\mathrm{d}\Delta/\mathrm{d}s|$ is $\bar{\Delta}/\xi_\beta$. We expect that for a region like this the contribution of each of the terms in (4) will be of the order of k_BT_c, namely, $a\xi_\beta\beta\bar{\Delta}^4 \sim a\gamma\hbar^2\bar{\Delta}^2/\xi_\beta \sim k_BT_c$. From here we obtain $\xi_\beta \sim (a\gamma^2\hbar^4/\beta k_BT_c)^{1/3}$, and using equation (1.28) in [10] we arrive at expression (3). The order parameter that we have used above is $\psi = \Delta/\bar{\Delta}$.

Since $\mathrm{d}\mathbf{A}/\mathrm{d}t$ is proportional to the electric field, the evolution of $\tilde{A}_j$ follows from Ohm's law, that can be written as

$$\frac{\mathrm{d}\tilde{A}_j}{\mathrm{d}t} = -\Gamma_{A,j}\frac{\partial G}{\partial\tilde{A}_j} \quad (5)$$

with $\Gamma_{A,j} = uDL/2N\xi_\beta^3 k_BT_c$, where in the case of a dirty material with no magnetic impurities $u = \pi^4/14\zeta(3) = 5.79$ [9,10].

The range of validity of TDGL is very limited [11]. A model that is valid as long as there is local equilibrium is due to Kramer and Watts–Tobin [12,13]. Writing $\psi_j = |\psi_j|\exp(\mathrm{i}\chi_j)$, this model assigns different relaxation rates to $|\psi_j|$ and to χ_j:

$$\frac{\mathrm{d}|\psi_j|}{\mathrm{d}t} = -\frac{\Gamma_\psi}{\sqrt{1+K|\psi_j|^2}}\frac{\partial G}{\partial|\psi_j|},$$
$$\frac{\mathrm{d}\chi_j}{\mathrm{d}t} = -\frac{\Gamma_\psi\sqrt{1+K|\psi_j|^2}}{|\psi_j|^2}\frac{\partial G}{\partial\chi_j}, \quad (6)$$

with $K \approx 15Dk_BT_c\tau_{\mathrm{ph}}^2/(\hbar\xi_\beta^2)$, where τ_{ph} is the electron-phonon inelastic scattering time.

The evolutions of the variables reviewed above all have the same structure: during a period of time τ that is short in comparison with the relaxation time, the variable x changes by $\Delta x = -\Gamma(\{x\})(\partial G/\partial x)\tau$, where x could stand for $\mathrm{Re}\psi_j$, $\mathrm{Im}\psi_j$, $\tilde{A}_j$, $|\psi_j|$ or χ_j, and $\{x\}$ could stand for several of these variables. This is true provided that fluctuations are ignored. When thermal fluctuations are taken into account, we have instead

$$\Delta x = -\Gamma(\{x\})\frac{\partial G}{\partial x}\tau + \eta, \quad (7)$$

where η is a random variable with appropriate distribution, called the 'Langevin term.' It can be shown that η has Gaussian distribution, with mean value and variance

$$\langle\eta\rangle = k_BT\Gamma\frac{\partial\log(J\Gamma)}{\partial x}\tau, \quad \langle\eta^2\rangle = 2k_BT\Gamma\tau, \quad (8)$$

where T is the local temperature, T_j, and J is the Jacobian of the variables in use. For instance, if we use the variables $|\psi_j|$ and χ_j, then $J = \partial(\mathrm{Re}\psi_j, \mathrm{Im}\psi_j)/\partial(|\psi_j|, \chi_j) = |\psi_j|$. An intuitive derivation of the distribution of η is provided in [14].

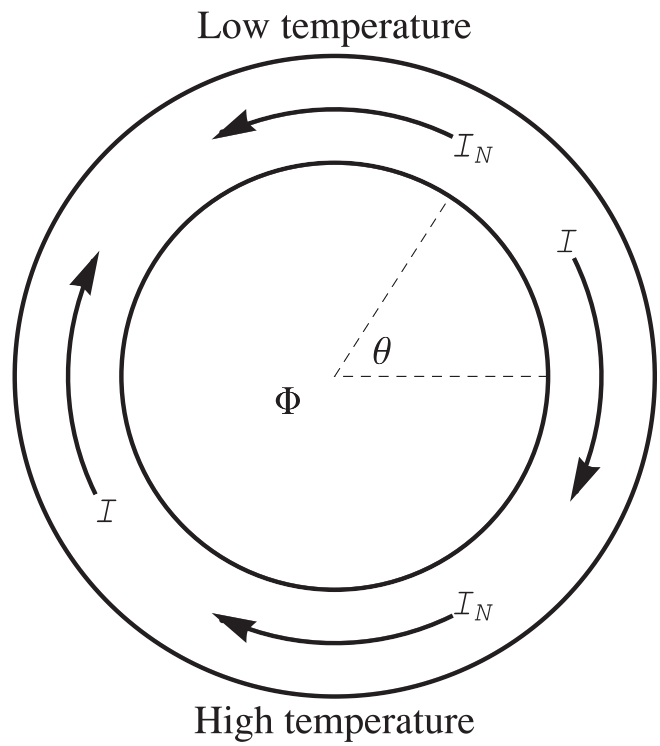

Fig. 1. (Reused from [18]) Superconducting ring that encloses a magnetic flux Φ. I is the total current around the ring and I_N is the normal current. One extreme of the ring is at high temperature $T_{\max}$ and the other at low temperature $T_{\min}$. θ measures the angle of any given position from a point at average temperature, at the right.

3. Flux-induced Nernst effect in rings

In the case of 1D samples there is no room for the usual Nernst effect, but we predict a qualitatively new effect for samples with ring topology: a thermoelectric voltage that is induced by the enclosed magnetic flux rather than by the magnetic field. We find that this voltage is largest in the direction perpendicular to the temperature gradient and is present when the magnetic flux is neither an integer nor a half-integer multiple of the quantum of flux Φ_0. Another thermoelectric phenomenon in rings is the appearance of magnetic flux in bimetallic loops [15,16].

We consider a superconducting ring with an average temperature close to T_c, that encloses a magnetic flux Φ, as shown in Fig. 1. For flux in the range $0 < \Phi < 0.5\Phi_0$, a diamagnetic current I flows around the ring [17]. In the region where the temperature is higher than the average, superconductivity is weaker than the average and therefore the super current will be smaller than the average; as a consequence, a normal current I_N will have to reinforce the supercurrent in order to reach the total current I. On the other hand, in the region of lower temperature, super current should be large and the normal current should oppose it. We therefore predict that, to maintain this normal current, a potential difference is required, higher in the region close to $\theta \approx 0$ in Fig. 1, and lower in the region close to $\theta \approx \pi$.

In order to check this prediction, we have evaluated the electrochemical potential, as a function of position, of the enclosed flux, and of ring parameters, as described in Section 2. This was done in [18] and here we summarize the results. We assumed for simplicity that the temperature is a linear function of position along the plane of the sample, so that as a function of the angle θ in Fig. 1,

$$T(\theta) = \frac{1}{2}[T_{\min}(1+\sin\theta) + T_{\max}(1-\sin\theta)], \quad (9)$$