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Spectral characteristics of the coherent dynamics of the order parameter in superconducting nanorods

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ABSTRACT

Within the density-matrix formalism based on the Bogoliubov-de Gennes equations approach we investigate the dynamics of the non-equilibrium BCS pairing induced by a sudden change of the coupling constant in quasi-1D and quasi-0D samples. We demonstrate that two different dynamical regimes of the amplitude of the BCS gap can be distinguished: an initially damped oscillation in the case of short quantum wires and purely irregular dynamics in the case of nanorods.

Spontaneous gauge symmetry breaking, i.e., the phenomenon in a physical system, where the symmetry of the vacuum is lower than the symmetry of the system Hamiltonian, results in two typical fundamental collective excitations: gapped amplitude/Higgs modes and gapless phase/Goldstone modes. The amplitude mode of the order parameter does not couple directly to any external probe (in the case of a clean system). It can only be excited by other excitations which shake the ground state [1,2]. If such other excitations are coupled to external probes, the amplitude mode appears by stealing weight from them (for an example in superconductivity see Ref. [3]). This was the reason why until recently the amplitude mode was not detected experimentally in superconducting and superfluid systems. This mode was recently identified in a 2D bosonic neutral superfluid close to a quantum phase transition to a Mott insulating phase. [4] There is a more crude way of shaking the condensate to observe the Higgs mode in superconductors. This is a pump probe experiment, in which a THz pulse is applied below the SC gap. The creation of a large number of excitations shake the condensate. [5–7] Only recently experimentalists have demonstrated the oscillatory dynamics of the BCS pairing induced by an intense monocycle-like THz pulse in a superconducting NbN film. [8]

Ultracold Fermi gases offer alternative possibilities to shake the condensate by almost instantaneously modifying the BCS pairing interaction. [9] This can be achieved by tuning an external magnetic field in the vicinity of a Feshbach resonance in particle scattering. Such an excitation allows for an investigation of the coherent pairing dynamics far away from equilibrium, which develops on a short time scale after an initial sudden perturbation shorter than any characteristic time scales in the system. [10–15] Such fast experiments can open up a possibility to induce “phase transitions” [16] and dynamics of order parameters in correlated materials, allowing the coherent fast manipulation of correlated systems. Furthermore, ultra-cold atomic gases offer a unique opportunity to explore the influence of confinement / dimensionality on the pairing correlations, [17,18,19,20] because dimensionality and confinement can be precisely controlled by tuning external parameters [17,21–23]. Restricting the dimensionality of Fermi gases may pave the way toward experimental evidences of unconventional phases, like the FFLO state, [24–26] etc.

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Unconventional characteristics have triggered the general interest in studies of superconducting and superfluid systems with reduced dimensionality. Fascinating in its own right, the field of superfluidity / superconductivity with quasi-low dimensional nature combined with the field of non-equilibrium physics may help to achieve control and manipulation over the pairing correlations in such systems. It is the purpose of this paper to explore the influence of the confinement strength on the pairing dynamics after an ultra-short perturbation $t_{\rm perturb} \ll \tau_\Delta \ll \tau_\epsilon$. Here the quasiparticle energy relaxation time at temperature T is, $\tau_\epsilon \sim \hbar E_F/\max(\Delta^2, T^2)$ with E_F being the Fermi energy and $\tau_\Delta \sim \hbar/|\Delta|$ is a typical dynamical time scale of the superconducting order parameter. Under such excitation conditions the quasiparticle spectrum looses its physical meaning and the system evolves non-adiabatically. The evolution of the system is collisionless in the time interval $t < \tau_\epsilon$. Within the density-matrix formalism based on the Bogoliubov–de Gennes equations we investigate two different dynamical regimes of the amplitude of the BCS gap. (cf. [6,7]) Here we consider metallic superconductors. We model the system confinement by a three-dimensional infinite box potential with the width (length) given by L_x and L_y (L_z). We use the parameters typical for Sn: $g_i N(0) = 0.25$ (g is the electron-electron interaction strength and $N(0)$ is the bulk energy density of states for one spin projection at the Fermi surface) and $T_D = \frac{\hbar\omega_D}{k_B} = 195\,\mathrm{K}$ (Debye temperature) corresponding to the Debye energy $\hbar\omega_D = 16.8$ meV.[1]

To start with we consider the Hartree-Fock-Bogoliubov Hamiltonian

$$H_{\rm HFB} = \int d\mathbf{r} \sum_\alpha \psi^\dagger_\alpha(\mathbf{r}) H_e \psi_\alpha(\mathbf{r}) + \Delta(\mathbf{r},t)\psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r}) + \Delta^*(\mathbf{r},t)\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})$$

with the order parameter $\Delta(\mathbf{r},t) = g\langle\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle$. Here $\psi_\alpha(\mathbf{r})$ and $\psi^\dagger_\alpha(\mathbf{r})$ are the field operators for an electron with spin α. $H_{\rm e}$ is the single-particle electron Hamiltonian, which takes into account confinement. A point-like form of the electron-electron interaction characterized by the coefficient g is assumed, and an appropriate momentum cut-off confining the interaction to a narrow layer, the Debye window, near the Fermi surface is implied. Making use of the canonical Bogoliubov transformation, which expresses the electron field operators in terms of new Fermi operators $\gamma_{p\uparrow\downarrow}$, $\gamma^\dagger_{p\uparrow\downarrow}$ as

$$\psi_\uparrow(\mathbf{r},t) = \sum_p \gamma_{p\uparrow}(t)u_p(\mathbf{r}) - \gamma^\dagger_{p\downarrow}(t)v^*_p(\mathbf{r}) \tag{1}$$

$$\psi_\downarrow(\mathbf{r},t) = \sum_p \gamma_{p\downarrow}(t)u_p(\mathbf{r}) + \gamma^\dagger_{p\uparrow}(t)v^*_p(\mathbf{r}), \tag{2}$$

the amplitudes $u_p(\mathbf{r})$, $v_p(\mathbf{r})$ satisfy the Bogoliubov-de Gennes equations at the initial time

$$\begin{pmatrix} H_e & \Delta^{(0)}(\mathbf{r}) \\ \Delta^{(0)*}(\mathbf{r}) & -H^*_e \end{pmatrix}\begin{pmatrix} u_p(\mathbf{r}) \\ v_p(\mathbf{r}) \end{pmatrix} = E_p \begin{pmatrix} u_p(\mathbf{r}) \\ v_p(\mathbf{r}) \end{pmatrix}. \tag{3}$$

Therefore, the Hamiltonian can be reduced to the diagonal form

$$H_{\rm HFB} = E_{\rm g} + \sum_{p,\sigma} E_p \gamma^\dagger_{p,\sigma}\gamma_{p,\sigma}, \tag{4}$$

as long as the current value of $\Delta(\vec{r})$ is equal to its initial value $\Delta^{(0)}(\vec{r})$. Here the constant E_g is the ground state energy of the superconductor and E_p can be understood as the excitation energy.

As in [6,7] we adopt the Anderson approximate solution for the BdG equations, i.e., we assume that

$$u_p(\mathbf{r}) = u_p\varphi_p(\mathbf{r}) \quad \text{and} \quad v_p(\mathbf{r}) = v_p\varphi_p(\mathbf{r}), \tag{5}$$

where $\varphi_p(\mathbf{r})$ are the single-electron wave functions in the confinement potential with energy ξ_p (relative to the Fermi energy). In this approximation the equilibrium state of the superconductor is characterized by the relations

$$u_p^2 - v_p^2 = \xi_p/E_p \quad \text{and} \quad 2u_p v_p = \Delta^{(0)}_{p,p}/E_p, \tag{6}$$

where $E_p = \sqrt{\xi_p^2 + \Delta^{(0)2}_{p,p}}$ and the matrix $\Delta^{(0)}_{p,q} = \int d\vec{r}\varphi^*_p(\mathbf{r})\Delta^{(0)}(\mathbf{r})\varphi_q(\mathbf{r})$. Due to the external perturbation the Hamiltonian acquires the following form

$$H_{\rm HFB} = \sum_{p,\sigma} R_p \gamma^\dagger_{p,\sigma}\gamma_{p,\sigma} + \sum_p \left[C_p\gamma^\dagger_{p\uparrow}\gamma^\dagger_{\bar{p}\downarrow} + C^*_p\gamma_{\bar{p}\downarrow}\gamma_{p\uparrow}\right]. \tag{7}$$

$\bar{p}$ indicates a time-reversed state, i.e., $\langle\vec{r}|\bar{p}\rangle = \langle p|\vec{r}\rangle$. The functions R_p and C_p depend on the initial and the current values of the order parameter as

$$R_p = \frac{\xi_p^2 + \Delta^{(0)}_{p,p}{\rm Re}[\Delta_{p,p}(t)]}{E_p}, \tag{8}$$

$$C_p = \frac{\xi_p}{E_p}\left\{{\rm Re}[\Delta_{p,p}(t)] - \Delta^{(0)}_{p,p}\right\} + i{\rm Im}[\Delta_{p,p}(t)]. \tag{9}$$

The order parameter is given in the Bogoliubov basis Eqs. (1) and (2)] by

$$\Delta_{p,p} = g\sum_{q,k_q} V_{q,p}\Big\{v_q^{*2}\left\langle\gamma^\dagger_{\bar{q}\downarrow}\gamma^\dagger_{q\uparrow}\right\rangle - u_q^2\left\langle\gamma_{q\uparrow}\gamma_{\bar{q}\downarrow}\right\rangle - u_q v^*_q\Big[\left\langle\gamma^\dagger_{q\uparrow}\gamma_{q\uparrow}\right\rangle + \left\langle\gamma^\dagger_{\bar{q}\downarrow}\gamma_{\bar{q}\downarrow}\right\rangle\Big] + u_q v^*_q\Big\}, \tag{10}$$

with the interaction matrix element

$$V_{q,p} = \int d\mathbf{r}\,|\varphi_q(\mathbf{r})|^2\,|\varphi_p(\mathbf{r})|^2. \tag{11}$$

Therefore, the pairing dynamics of a superconductor as manifested in the time evolution of the order parameter $\Delta(\mathbf{r}, t)$ is governed by the time evolution of the four expectation values $\langle\gamma_{p\uparrow}(t)\gamma_{q\downarrow}(t)\rangle$, $\langle\gamma^\dagger_{q\uparrow}(t)\gamma_{p\uparrow}(t)\rangle$, $\langle\gamma^\dagger_{p\downarrow}(t)\gamma_{q\downarrow}(t)\rangle$ and $\langle\gamma^\dagger_{p\downarrow}(t)\gamma^\dagger_{q\uparrow}(t)\rangle$. We set up and numerically solve the Heisenberg equations of motion for these dynamical variables. In doing so, we assume that before the non-adiabatic perturbation the superconducting system has been in the ground state, which is the quasiparticle vacuum. This means that all four correlators are zero and $\Delta_{p,p} = \Delta^{(0)}_{p,p}$. However, due to the interaction quench finite correlators are excited and thus $\Delta_{p,p} \neq \Delta^{(0)}_{p,p}$.

In this paper we analyze the amplitude dynamics of the spatially averaged BCS gap

$$\bar{\Delta}(t) = \int_V d^3r\,\Delta(\mathbf{r},t)$$

of a BCS superconductor of volume $V = L_x L_y L_z$ after an interaction quench from $g_i N(0) = 0.25$ to $g_f N(0) = 0.24$ which models an ultrafast laser excitation [7]. We start with the dynamics of short quantum wires. Then, we investigate the transition from an initially damped amplitude oscillation of the gap known from quantum wires [7] to purely irregular dynamics in the case of nanorods. We will explain this transition on the basis of a set of linearized equations of motion and by that, we will link the characteristics of the two regimes to the distribution of the quasi-particle energies E_p. To do so, we focus on a system with the width $L_x = L_y = 4.3$ nm, which is narrow enough to exhibit strong quantization effects but is still in the regime accessible for experiment.

Fig. 1 shows the amplitude dynamics of the BCS gap for such a system with the length given by $L_z = 300$ nm (upper, blue curve), i.e., a short quantum wire. One clearly observes an initially damped oscillation of the gap with one main frequency which is the main dynamical characteristic of a quantum wire (cf. [7]). However, after

[1] For the systems investigated in this work $E_F \approx 10.7$ eV and $\Delta \sim 0.5$ meV, i.e., $\tau_\Delta \sim 1.3$ ps and $t_\epsilon \sim 14$ ns.