ARTICLE IN PRESS
JID: PHYSC [m5G;June 21, 2016;14:15]
Physica C: Superconductivity and its applications 000 (2016) 1–7
Contents lists available at ScienceDirect
Physica C: Superconductivity and its applications
journal homepage: www.elsevier.com/locate/physc

Magnetic flux distributions in chiral helimagnet/superconductor bilayers

Masaru Kato[a,*], Saoto Fukui[a], Osamu Sato[b], Yoshihiko Togawa[c]

[a] Department of Mathematical Sciences, Osaka Prefecture University, 1-1, Gakuencho, Nakaku, Sakai, Osaka 599-8531, Japan
[b] Osaka Prefecture University College of Technology, 26-12, Saiwaicho, Neyagawa, Osaka 572-8572, Japan
[c] Department of Physics and Electronics, Osaka Prefecture University, 1-1, Gakuencho, Nakaku, Sakai, Osaka 599-8531, Japan

ARTICLE INFO

Article history:
Received 11 January 2016
Revised 25 March 2016
Accepted 7 June 2016
Available online xxx

Keywords:
Vortex states
Chiral helimagnet
Ginzburg–Landau equation
Finite element method

ABSTRACT

Vortex states in a chiral helimagnet/superconductor bilayer are investigated numerically, using the Ginzburg–Landau equations with the finite element method. In this bilayer, effect of the chiral helimagnet on the superconductor is taken as an external field. Magnetic field distribution can be controlled by an applied field to the bilayer. It is shown that a single vortex in a gradient field is elongated along the field gradient. In zero applied field, there are up- and down vortices which are parallel or antiparallel to the z-axis, respectively. But increasing the applied field, down-vortices disappear and up-vortices form undulated triangular lattices.

1. Introduction

Interplay between magnets and superconductors were studied for decades [1]. For example, superconductors with homogenous ferromagnet [2] and artificial magnetic dots [3–5] were studied. It was pointed out that in a ferromagnet/superconductor bilayer system, a vortex state appears with magnetic domain wall [1]. Such domain wall structure was studied theoretically and experimentally [6–8].

In contrast to previous studies, we study a chiral helimagnet/superconductor bilayer system (Fig. 1). In the chiral helimagnet, magnetic moments show helical rotation [9]. Such magnetic structure in $Cr_{1/3}NbS_2$ was observed by Togawa et al. using Lorenz microscopy [10]. They also observed chiral magnetic soliton lattices under an external field, of which period agrees with theory [9].

In the ferromagnet/superconductor bilayer with domain structures, the magnetic field from the ferromagnet is H or $-H$. But magnetic field from the chiral helimagnet changes continuously. Therefore, vortex states are expected to be different from those in ferromagnet/superconductor bilayers. In this study, we focus on such effects of continuously changing magnetic field onto the vortex states in the superconductor. In order to obtain vortex states in the chiral helimagnet/superconductor bilayer, we use phenomenological Ginzburg-Landau equations.

In Section 2, we show our models and numerical method. In Section 3, numerical results about the vortex states in gradient field are given. In Section 4, we show stable vortex states in chiral helimagnet/superconductor bilayer. Section 5 is devoted to conclusions.

2. Models and method

We consider a two dimensional superconducting system. We take into account of the effect of chiral helimagnet onto the superconductor as an external field, which oscillates spatially. Generally, a superconductor in a varying external field can be treated by the Ginzburg–Landau (GL) equations.

We start from the GL free-energy,

$$\mathcal{F}(\psi\cdot\mathbf{A}) = \int d\Omega\left[\frac{1}{2}\left(\sqrt{\beta}|\psi|^2+\frac{\alpha}{\sqrt{\beta}}\right)^2 + \frac{1}{4m}\left|\left(-i\hbar\nabla+\frac{2e}{c}\mathbf{A}\right)\psi\right|^2+\frac{|\mathbf{h}-\mathbf{H}|^2}{8\pi}+\frac{1}{8\pi}(\mathrm{div}\mathbf{A})^2\right]. \tag{1}$$

Here ψ is the superconducting order parameter, $\mathbf{A}$ is a magnetic vector potential and $\mathbf{h}=\nabla\times\mathbf{A}$. Spatially varying external magnetic field is $\mathbf{H}(\mathbf{r})$ and $\alpha = a(T/T_c - 1)$ and β are constants for the GL theory. The term $\frac{1}{8\pi}(\mathrm{div}\mathbf{A})^2$ is added to insure the London Gauge $\mathrm{div}\mathbf{A}=0$. In order to minimize this free-energy with respect to ψ and $\mathbf{A}$, we use the finite element method (FEM) [11]. In the FEM,

* Corresponding author.
E-mail address: kato@ms.osakafu-u.ac.jp (M. Kato).

http://dx.doi.org/10.1016/j.physc.2016.06.004
0921-4534/

Please cite this article as: M. Kato et al., Magnetic flux distributions in chiral helimagnet/superconductor bilayers, Physica C: Superconductivity and its applications (2016), http://dx.doi.org/10.1016/j.physc.2016.06.004

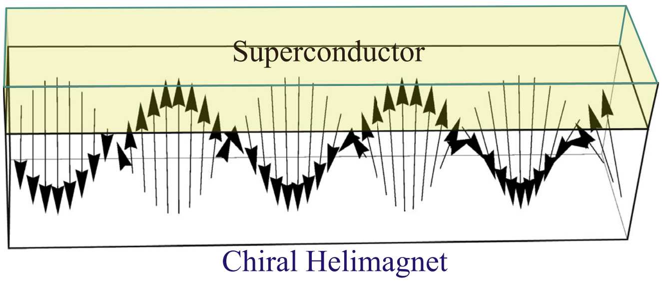

Fig. 1. Schematic diagram of a superconductor/chiral helimagnet double layer system.

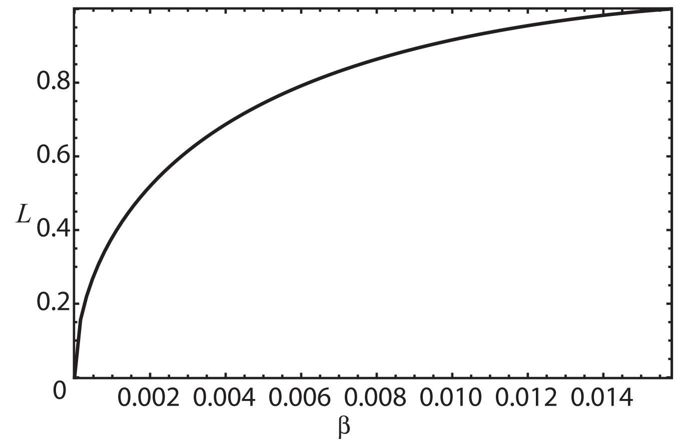

Fig. 2. The modulus k of the Jacobi's elliptic function in Eq. (18) as a function of the reduced applied field $\beta = 2\mu_B H_{appl}/\mathcal{J}S$.

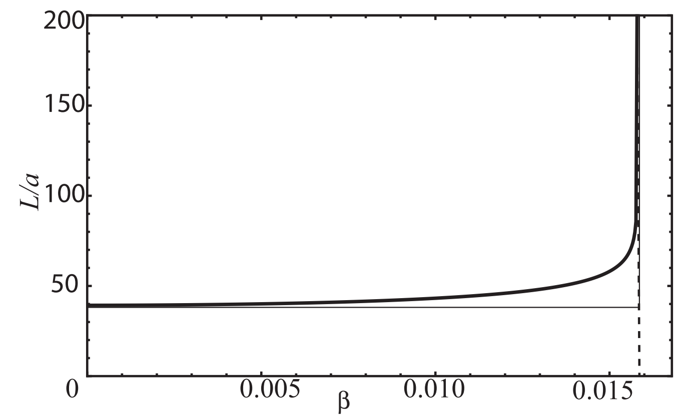

Fig. 3. The period of the chiral helimagnet as a function of a function of the reduced applied field $\beta = 2\mu_B H_{appl}/\mathcal{J}S$.

we use the Galarkin method, in which we use the Frechét derivative. The Frechét derivatives of the free-energy about $\tilde{\psi}$ and $\tilde{\mathbf{A}}$ becomes

$$\int d\Omega\Big[(i\nabla\psi - \mathbf{A}\psi)(-i\nabla\tilde{\psi}^* - \mathbf{A}\tilde{\psi}^*) + (i\nabla\tilde{\psi} - \mathbf{A}\tilde{\psi})(-i\nabla\psi^* - \mathbf{A}\psi^*) + \frac{1}{\xi(T)^2}(|\psi^*| - 1)(\psi\tilde{\psi}^* + \tilde{\psi}\psi^*)\Big] = 0 \tag{2}$$

$$\int d\Omega\Big[\kappa^2\xi(T)^2\{\mathrm{div}\mathbf{A}\mathrm{div}\tilde{\mathbf{A}} + (\nabla\times\mathbf{A})(\nabla\times\tilde{\mathbf{A}})\} + |\psi|^2\mathbf{A}\cdot\tilde{\mathbf{A}} - \frac{i}{2}(\psi^*\nabla\psi - \psi\nabla\psi^*)\tilde{\mathbf{A}}\Big] = \kappa^2\xi(T)^2\int d\Omega\frac{2\pi}{\Phi_0}\mathbf{H}\cdot\left(\nabla\times\tilde{\mathbf{A}}\right) \tag{3}$$

where ξ, κ and λ are the coherence length, the GL parameter and the penetration depth, respectively. $\Phi_0 = hc/2e$ is the flux quanta. In the FEM, the system is divided into finite elements, and in 2-dimensional system, we use triangular elements. (Fig. 1) All physical quantities are expanded by the aria coordinates. There are three area coordinates for e-th element

$$N_i(x, y) = \frac{1}{2S_e}(a_i + b_i x + c_i y) \quad (i = 1, 2, 3) \tag{4}$$

where coefficients are given as,

$$a_i = x_j - x_k \tag{5}$$

$$b_i = y_j - y_k \tag{6}$$

$$c_i = x_j - x_k \tag{7}$$

Here, (i, j, k) is a cyclic permutation of (1, 2, 3) and (x_i, y_i, z_i) is the coordinate of i-th node of the e-th element. Using the area coordinates, the order parameter and the vector potential are expanded as,

$$\psi(x, y) = N_1(x, y)\psi_1 + N_2(x, y)\psi_2 + N_3(x, y)\psi_3 \tag{8}$$

$$\mathbf{A}(x, y) = N_1(x, y)\mathbf{A}_1 + N_{21}(x, y)\mathbf{A}_2 + N_3(x, y)\mathbf{A}_3 \tag{9}$$

inside of eth element. Here ψ_i and $\mathbf{A}_i$ are values of the order parameter and the vector potential at ith node. We substitute these expansions into Eqs. (2) and (3), and set the test function as

$$\tilde{\psi} = N_i(x, y) \quad (i = 1, 2, 3) \tag{10}$$

$$\tilde{\mathbf{A}} = N_i(x, y)\mathbf{e}_j \qquad (i = 1, 2, 3;\ j = x, y) \tag{11}$$

Then we get the GL equations in the FEM as,

$$\sum_j\big[P_{ij}(\{\mathbf{A}\}) + P_{ij}^{2R}(\{\psi\})\big]\mathrm{Re}\psi_j + \sum_j[Q_{ij}(\{\mathbf{A}\}) + Q_{ij}^2(\{\psi\})]\mathrm{Im}\psi_j = V_i^R(\{\psi\}) \tag{12}$$

$$\sum_j\big[-Q_{ij}(\{\mathbf{A}\}) + Q_{ij}^2(\{\psi\})\big]\mathrm{Re}\psi_j + \sum_j\big[P_{ij}(\{\mathbf{A}\}) + P_{ij}^{2I}(\{\psi\})\big]\mathrm{Im}\psi_j = V_i^I(\{\psi\}) \tag{13}$$

$$\sum_j R_{ij}(\{\psi\})A_{jx} + \sum_j S_{ij}A_{jy} = T_i^x(\{\psi\}) - U_i^y \tag{14}$$

$$-\sum_j S_{ij}A_{jx} + \sum_j R_{ij}(\{\psi\})A_{jy} = T_i^y(\{\psi\}) + U_i^x \tag{15}$$

Here definitions of the coefficients are given in Appendix.

For a given spatially varying field $\mathbf{H}(\mathbf{r})$, we solve Eqs. (12)–(15) numerically and obtain a stable vortex state.

For the helimagnet/superconductor bilayer, we assume the external field onto the superconductor is proportional to the magnetization of z-component of the magnetization of the chiral helimagnet. The magnetization of chiral helimagnet can be derived as follows. ([9])

The Hamiltonian for spin in the chiral helimagnet is given as,

$$\mathcal{H} = -J\sum_i \mathbf{S}_i\cdot\mathbf{S}_{i+1} + \mathbf{D}\cdot\sum \mathbf{S}_i\times\mathbf{S}_{i+1} - 2\mu_B H_{\mathrm{appl}}\sum_i S_{iz} \tag{16}$$

Here the first term is ferromagnetic exchange interaction, where $J(\ > 0)$ is the exchange energy. The second term is the