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Using measurements of the spatial SNR to optimize phase contrast X-ray imaging

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A B S T R A C T

X-ray phase contrast imaging is a measurement task which is challenging to optimize, because many physical effects determine signal and noise. If we describe the detail visibility by the spatial signal to noise ratio, SNR(u), we can optimize an imaging setup by maximizing its SNR(u). We propose a measurement method for the spatial SNR which is suitable for this purpose. It consists of measuring a series of images from which the spatial SNR is calculated. This allows a convenient and exact optimization of the SNR that does not rely on theoretical simplifications and is not specific to X-ray imaging. We demonstrate the measurement method for the example of choosing the optimal geometrical magnification for cone-beam inline X-ray phase contrast. Additionally, we propose the use of a known signal reconstruction method – the Wiener Deconvolution – to improve the detail visibility by post-processing images within the limits given by the measured SNR(u). As the SNR(u) gives the degree of this improvement, we derive a measure for the effective spatial resolution from the SNR(u).

1. Introduction

The spatial SNR (also called frequency-dependent SNR) describes the performance of an imaging setup, specifically if image details of a certain size are visible in a noisy image. It includes image blur effects, commonly described with a modulation transfer function (MTF), and is thus more general than the temporal SNR (not frequency-dependent).

In the case of X-ray imaging, the formation of an image is a combined process which includes effects which either decrease or increase the SNR and which depend on the experimental setup. Examples for effects that decrease SNR are image blur or polychromatic effects [1]. An effect that increases the SNR is (inline) phase contrast [2–5]. The signal strength can also depend strongly on the measurement conditions, e.g. X-ray energy.

One approach for optimizing an imaging setup for high SNR is to simulate all effects on the SNR. If used to characterize an actual experimental setup, this approach requires the properties of the experimental setup to be accurately determined and modeled correctly. An example of a problematic model assumption is a Gaussian form for the MTF of the source or detector, because actual MTFs can differ greatly from this form.

In this work we approach the problem differently: we measure the spatial SNR directly and build a feedback loop that allows us to optimize the measurement parameters. This of course only works for parameters that can easily be varied in an existing experimental setup, for example geometrical magnification or tube voltage/filters (source spectrum). The optimum is also specific to the imaging task, for example material composition and thickness of the sample, and the size of the details to be resolved—although similar tasks will have similar optima.

One of the problems in optimizing the experimental parameters of a laboratory (cone-beam) inline phase contrast imaging setup is the determination of the optimal geometrical X-ray magnification [6]. Usually the optimal magnifications for the phase contrast and resolution (MTF) differ, we then need to find the magnification at which the combination of both effects leads to an optimal SNR. Because we analyze the signal gain via the power spectrum, the gain in SNR can theoretically be predicted from the derivations of Fourier filter phase retrieval algorithms, for example [7].

If we measure the SNR at different sample positions, we can directly determine the optimal position without the need to model the imaging setup. In a setup without phase contrast, the same procedure yields the position at which the spatial resolution is optimal.

To understand the concepts used here, basic knowledge of the subjects of stochastic processes [8], Bayesian probability theory [9] and Fourier signal processing [10] is useful.

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2. Measuring signal and noise

The aim of the SNR measurement method we present here is not primarily to derive an absolute measure of imaging performance. Because the absolute value of the SNR depends on how much signal/structure is on average in the measurement region, the absolute SNR values are specific to the imaging task and sample. What we aim to do is to compare SNR measurements for a representative sample under different measurement conditions. This way, we can find a optimum of the imaging performance that is also valid for similar imaging tasks.

To make clear what we mean by SNR, we define two types of noise, which then define corresponding types of SNR:

- Temporal noise: (Statistical) deviations between independent measurements under identical conditions for one physical variable (e.g. intensity in one pixel), usually at different times. It can be reliably measured under any conditions that do not change in time and for processes that are Markov.
- Spatial noise: Temporal noise at different points in space (e.g. pixels in an area detector). This definition is both exact and practically useful, because temporal noise can be determined reliably.

Noise can have spatial correlations, signals almost always have – but pixels are assumed to be independent if temporal noise is considered in imaging.

2.1. Measuring temporal SNR

Temporal noise can be reliably measured even if a sample is present in the imaging setup. This is done by measuring a series of K images $d_j(x)$ under identical conditions and subtracting the average $s_{K\setminus j}(x)$ of all other images from the individual images to get noise images $n_j(x)$:

$$n_j(x) = d_j(x) - s_{K\setminus j}(x) = d_j(x) - \sum_{m=1,m\neq j}^{K} \frac{d_m(x)}{K-1}. \quad (1)$$

Here x is the n-dimensional spatial coordinate. A similar approach has been described in [11,12].

If we additionally approximate the signal as the average over all images, we can determine the temporal SNR per pixel as:

$$\mathrm{SNR}_{\mathrm{temp}}(x) = \frac{\mathrm{Avg}_j(d_j(x))^2}{\mathrm{Var}_j(n_j(x))}. \quad (2)$$

2.2. Measuring spatial SNR

To determine if details in a measured image can be resolved, the frequency-dependent or spatial SNR(u) must be used instead of the temporal SNR, because the temporal SNR does not include modulation transfer effects such as image blur (MTF) or phase contrast.

For example if we digitally apply a Fourier filter blur to an image, the temporal SNR is increased. The spatial SNR is unchanged—there is no information loss, because the filter is invertible. Similarly, an additional phase contrast signal does not influence the temporal SNR but it does increase the spatial SNR.

To define signal and noise, we use a linear additive data model with data $d(x)$ (e.g. number of detected photons), signal $s(x)$ and noise $n(x)$ at a point x on the detector:

$$d(x) = s(x) + n(x) \quad (3)$$

where $d(x)$ and $n(x)$ describe random processes, while $s(x)$ is deterministic. We define the spatial SNR as

$$\mathrm{SNR}(u) = \frac{S(u)}{N(u)}, \quad (4)$$

where $S(u)$ and $N(u)$ denote power spectra of $s(x)$ and $n(x)$. See Appendix A.1 for some general properties of power spectra. This definition is different from e.g. the one used in [13], because we include effects such as phase contrast or different strengths of the absorption signal. It stems from the SNR being proportional to the signal power spectrum. This is necessary to characterize the SNR for a specific sample, therefore its value cannot be sample-independent.

To determine the SNR(u), we use a series of images of the same signal. This means that the images are taken with the same sample under identical conditions, including identical integration times τ for the detector. Let $\{d_j(x)\}$ be the series of images and

$$d_{\mathrm{avg}}(x) = \frac{1}{K}\sum_{j=1}^{K} d_j(x) \quad (5)$$

the average of the series. The power spectrum of an arbitrary image d satisfying Eq. (3) is

$$D(u) = |\mathcal{F}\{d\}(u)|^2 = S(u) + N(u) \quad (6)$$

because signal and noise are uncorrelated, see Appendix A.2 for a derivation. We will denote the power spectrum of the d_j as D_τ and the power spectrum of d_{avg} as D_{avg} and get

$$D_\tau(u) = S_\tau(u) + N_\tau(u) \quad (7)$$

$$D_{\mathrm{avg}}(u) = S_\tau(u) + \frac{1}{K^2}\sum_{j=1}^{K} N_\tau(u) = S_\tau(u) + \frac{1}{K}N_\tau(u) \quad (8)$$

because noise is uncorrelated with any other signal (including other noise). Note that in real space, noise has an average of zero and the positive and negative contributions from a noise sum cancel each other out partially.

The values used for $D_\tau(u)$ should be calculated as an average of the power spectra calculated for the individual d_j to reduce statistical errors. Then $D_{\mathrm{avg}}(u)$ is the power spectrum of the real space average of the images and $\langle D_\tau(u)\rangle$ is the average power spectrum of the same images. We can solve Eqs. (7) and (8) for signal and noise power spectra as functions of detected images

$$S_\tau(u) = \frac{D_{\mathrm{avg}}(u) - \langle D_\tau(u)\rangle K^{-1}}{\left(1-K^{-1}\right)} \quad (9)$$

$$N_\tau(u) = \frac{\langle D_\tau(u)\rangle - D_{\mathrm{avg}}(u)}{\left(1-K^{-1}\right)}. \quad (10)$$

These power spectra can also be of interest for certain evaluations and we can calculate the spatial SNR from them as

$$\mathrm{SNR}_\tau(u) = \frac{S_\tau(u)}{N_\tau(u)} = \frac{D_{\mathrm{avg}}(u) - \langle D_j(u)\rangle K^{-1}}{\langle D_j(u)\rangle - D_{\mathrm{avg}}(u)} \quad (11)$$

which can be used to calculate the SNR(u) from a series of measured images. We propose three applications for this measurement method:

- Optimization of an imaging setup with direct quantitative feedback. This could also be automated, e.g. for choosing the optimal setup geometry or source spectrum.
- Determination of the integration time required to resolve details of a certain size in an image (see Section 2.4).
- Noise suppression that uses the SNR, especially a Wiener Filter (see Section 3).

If the final result of the measurement is derived through an arbitrary transformation (e.g. negative logarithm) on an image, this method can be extended to calculate the SNR of the final result.

It also be extended to cases in which a final result image is calculated from a series of images (e.g. projections for a computed tomography). In this case, SNR images under identical conditions must be measured for every image in the series. If a final result r is calculated from M