Nuclear Inst. and Methods in Physics Research, A 875 (2017) 141–145

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima

A critical analysis of the technique of spin tune mapping in storage rings

S.R. Mane

Convergent Computing Inc., P. O. Box 561, Shoreham, NY 11786, USA

ARTICLE INFO

Keywords:
Polarized beams
Storage rings
Polarimetry
Electric and magnetic moments

ABSTRACT

A paper has recently been published which describes the technique of so-called 'spin tune mapping' to measure the 'stable spin axis' (spin closed orbit) of a spin polarized beam circulating in a storage ring. This paper presents an independent analysis of the technique, and significantly different findings are reported below. In particular, it is derived that there are several unquantified systematic errors which are not treated in the previous analysis.

1. Introduction

A paper has recently been published [1] which describes the technique of so-called 'spin tune mapping' to measure the 'stable spin axis' of a spin polarized beam circulating in a storage ring. The authors employ nonstandard notation and terminology in [1]: the 'stable spin axis' is the spin closed orbit or the rotation axis of the one-turn spin map on the closed orbit of the ring. A review of spin dynamics in accelerators can be found in [2]. It is claimed in [1] that the spin tune mapping technique can determine the orientation of the stable spin axis to microradian accuracy. Note, however, that the components of the polarization vector were not measured directly in [1]. Instead, the direction of the stable spin axis was deduced via measurements of the spin tune and the application of a theoretical model.

I have independently examined the analysis in [1] and my findings differ from the claims made in [1]. In addition to identifying various errors of algebra, I found there are several unquantified systematic errors which are not treated in the analysis in [1]. Numerous priority claims are also made in [1]. I comment on some of those priority claims and supply references to prior work in the literature [3–9].

This paper is organized as follows. Section 2 presents the basic notation and definitions. The spin maps for relevant beamlines and ring elements, which are pertinent to the analysis, are shown in Section 3. The solution for the spin tune is derived in Section 4. Differences with the formulas in [1] are pointed out. Section 5 presents the exact solution of an idealized model. It is shown that the solution derived in Section 4 agrees with the exact solution, up to terms of the first order in small quantities. However, the formulas derived in [1] do not agree with the exact model, even for the first order terms. In particular, for the scenario studied in [1], the determination of the radial component of the stable spin axis is subject to large uncertainties, which are not accounted for in the analysis in [1]. Section 6 comments on some of the priority claims made in [1] and describes prior work on the subject. Section 7 concludes.

2. Basic notation and definitions

We refer the reader to [2] for a review of spin dynamics in accelerators, including the electric dipole moment (EDM). We treat a particle of mass m and charge e, with velocity $\vec{v} = \vec{\beta}c$ and Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. The canonical particle coordinate and conjugate momentum are denoted by $\vec{r}$ and $\vec{p}$, respectively. Most of the analysis in this paper employs coordinate-free notation. Where explicit vector components are required, we follow [1] and employ the (right-handed orthonormal) basis vectors $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$, respectively radial (outward), vertical (up) and longitudinal (along the ring reference axis). We denote the spin vector by $\vec{s}$, treated as a semiclassical unit vector, with magnetic moment anomaly $G = (g-2)/2$. We treat the vector polarization only, and denote the polarization vector by $\vec{P}$. Neglecting the EDM,[1] the spin precession equation of motion in the externally prescribed electric and magnetic fields of the accelerator ($\vec{E}$ and $\vec{B}$, respectively) is given by the Thomas–BMT (Bargmann–Michel–Telegdi) equation [11,12]

$$\frac{d\vec{s}}{dt} = -\frac{e}{mc}\left[\left(G+\frac{1}{\gamma}\right)\vec{B} - \frac{G\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]\times\vec{s}\,. \tag{2.1}$$

Radiation fields are ignored and we treat nonradiatively polarized beams only. In this paper, the spin state of a particle is parameterized by a two-component spinor and the spin map through a beamline is

[1] The semiclassical relativistic spin precession equation including EDM terms is given in [10]. See also the review [2].

E-mail address: srmane001@gmail.com.

http://dx.doi.org/10.1016/j.nima.2017.09.026
Received 5 August 2017; Accepted 13 September 2017
Available online 20 September 2017
0168-9002/

parameterized using a 2×2 SU(2) matrix. The one-turn spin map on the closed orbit is conventionally written as follows[2]

$$M_{\rm OTM} = \exp\{-i\pi\nu\vec{\sigma}\cdot\vec{n}_0\}\,. \tag{2.2}$$

Here ν is the spin tune and $\vec{\sigma}$ is a vector of Pauli matrices. The rotation axis of the one-turn spin map on the closed orbit is denoted by the unit vector $\vec{n}_0$ (this vector is also known as the 'spin closed orbit'). For off-axis motion, the quantization axis of the spin eigenstates is denoted by $\vec{n}(\vec{r},\vec{p})$, which is a function of the orbital phase space [13]. Throughout most of this paper, we shall restrict attention only to motion on the closed orbit. For a steady state spin polarized beam circulating in a storage ring, i.e. after transients have decohered, the polarization vector $\vec{P}$ is parallel to the average $\vec{P} \parallel \langle\vec{n}\rangle$, where the average is taken over the orbital phase space. In general, this average is almost parallel to $\vec{n}_0$, and this approximation will be employed below. (See [14, Sec. (3.3)] for a model example where $\langle\vec{n}\rangle$ is not parallel to $\vec{n}_0$.)

The model treated in [1] was a racetrack ring, with a solenoid in each of the diametrically opposed straight sections. Optically, the two arcs (also the straight sections) were identical except for lattice imperfections. The two solenoids were treated as localized zero-length perturbations. We shall treat the above model in this paper. The spin tune in the unperturbed ring (no solenoids) was denoted by ν_s^0 and in the full ring by ν_s. The authors called ν_s^0 the 'unperturbed spin tune' [1]. The authors also employed the notation $\vec{c}$ instead of $\vec{n}_0$ for the spin closed orbit of the unperturbed ring and called it the 'stable spin axis.' It was assumed in [1] that the polarization vector in the unperturbed ring (after transients had decohered) points along $\vec{c}$.

Note that the authors in [1] claimed to measure the direction of the stable spin axis but they did *not* measure the components of the polarization vector directly. Instead they determined the values of two parameters a_+ and a_- [1, eq. (31)] where it was stated 'Consequently, the determination of $a_\pm$ amounts to the determination of the projections of the stable spin axis $\vec{c}$ onto a plane spanned by the vectors $\vec{n}_1$ and $\vec{n}_2^{\,\rm r}$.' (The vectors $\vec{n}_1$ and $\vec{n}_2^{\,\rm r}$ will be defined below.) A beam dynamics study using stored polarized deuterons was performed at COSY using two electron cooler solenoids in diametrically opposed straight sections. The authors generated artificial longitudinal 'imperfection fields' using the electron cooler solenoids. The formalism in [1] presents an analysis of the data from that experiment. The experimentally measured quantity was the spin tune. The authors employed the term 'spin tune jump' to refer to the change in the spin tune $\Delta\nu_s = \nu_s - \nu_s^0$, with the solenoids on and off. The direction of the stable spin axis was therefore *deduced* via measurements of the spin tune jump and a theoretical model.

3. Spin maps

The analysis below treats only rings where the spin closed orbit is vertical everywhere, in the ideal design, and the ring has no Siberian Snakes or spin rotators. See [15] for a review of Siberian Snakes and spin rotators in storage rings. COSY is an example of such a ring. We shall mostly employ the notation in [1] for ease of reference to make contact with their analysis. Note, however, that their notation does not follow the standard practice in the field. The origin was placed just before the first solenoid. The one-turn spin map is, with an obvious notation [1, eq. (21)]

$$M_{\rm OTM} = M_{\rm A_2}M_{\rm S_2}M_{\rm A_1}M_{\rm S_1}\,. \tag{3.1}$$

Here the term 'arc' includes the straight sections (lattice imperfections in the straight sections can tilt the spin closed orbit away from the vertical). The one-turn spin map of the unperturbed ring (i.e. without solenoids) is parameterized via

$$M_{\rm R} = M_{\rm A_2}M_{\rm A_1} = \exp\{-i\pi\nu_s^0\vec{\sigma}\cdot\vec{c}\}\,. \tag{3.2}$$

See [1, eq. (17)] and Eq. (2.2) above. The spin map of each arc is parameterized via [1, eq. (24)]

$$M_{\rm A_j} = \exp\Big\{-\frac{i}{2}\,\theta_j(\vec{\sigma}\cdot\vec{m}_j)\Big\} \qquad (j=1,2)\,. \tag{3.3}$$

Here θ_j is the spin rotation angle and $\vec{m}_j$ is the spin rotation axis of the spin map for each arc. It is assumed that $\vec{c}$ is almost but not exactly vertical. (It would be exactly vertical in the absence of lattice imperfections). The arcs are almost but not exactly identical (i.e. they would be exactly identical in the absence of lattice imperfections). Hence $\vec{m}_1$ and $\vec{m}_2$ are both nearly vertical (but they are not assumed to be equal). Also $\theta_1 \simeq \pi\nu_s^0$ and $\theta_2 \simeq \pi\nu_s^0$ (but they are not assumed to be equal). The spin map of each solenoid is parameterized via [1, eq. (25)]

$$M_{\rm S_j} = \exp\Big\{-\frac{i}{2}\,\chi_j(\vec{\sigma}\cdot\vec{n}_j)\Big\} \qquad (j=1,2)\,. \tag{3.4}$$

Here χ_j is the spin rotation angle and $\vec{n}_j$ is the spin rotation axis of each solenoid.[3] The solenoids are treated as zero length elements. The vectors $\vec{n}_1$ and $\vec{n}_2$ are nearly longitudinal (along the reference axis of the ideal ring) but they are not assumed to be exactly equal. The angles χ_1 and χ_2 were variable parameters in the analysis in [1]. In addition let us define [1, eq. (31)]

$$\chi_\pm = \frac{\chi_1 \pm \chi_2}{2}\,. \tag{3.5}$$

In addition to $\vec{n}_1$ and $\vec{n}_2$, the authors also employed a vector $\vec{n}_2^{\,\rm r}$ defined via [1, eq. (26)]

$$M_{\rm A_1}^{-1}M_{\rm S_2}M_{\rm A_1} \equiv \exp\Big\{-\frac{i}{2}\,\chi_2(\vec{\sigma}\cdot\vec{n}_2^{\,\rm r})\Big\}\,. \tag{3.6}$$

It is given by [1, eq. (27)]

$$\vec{n}_2^{\,\rm r} = \cos\theta_1\,\vec{n}_2 + \sin\theta_1\,(\vec{n}_2\times\vec{m}_1) + (1-\cos\theta_1)(\vec{m}_1\cdot\vec{n}_2)\vec{m}_1\,. \tag{3.7}$$

The authors then defined the spin map of the 'combined artificial imperfection' via $M_{\rm AI} = M_{\rm A_1}^{-1}M_{\rm S_2}M_{\rm A_1}M_{\rm S_1}$ [1, eqs. (23) and (28)]. The full one-turn spin map is then given by $M_{\rm OTM} = M_{\rm R}M_{\rm AI}$. The authors also defined the two variables [1, eq. (31)]

$$a_\pm = \vec{c}\cdot\vec{n}_2^{\,\rm r} \pm \vec{c}\cdot\vec{n}_1\,. \tag{3.8}$$

The above expressions are all in coordinate-free notation. The authors then made various approximations, using a coordinate basis, to derive the approximate expressions up to the first order in small quantities [1, eq. (78)]

$$a_\pm \simeq \cos(\pi\nu_s^0)c_z - \sin(\pi\nu_s^0)c_x \pm c_z\,. \tag{3.9}$$

For later use, I shall define the two parameters

$$\alpha_\pm = \vec{c}\cdot\vec{n}_2^{\,\rm r} \pm \vec{c}\cdot\vec{n}_1\,. \tag{3.10}$$

In coordinate-free notation, these are the same as $a_\pm$ in Eq. (3.8). However, when expanded in components, I shall show their values are different from those in Eq. (3.9). The matter will be treated below.

4. Spin tune

The spin tune of the full ring (with solenoids) is obtained from the parameterizations of the spin maps above and is obtained via $\cos(\pi\nu_s) =$

[2] The closed orbit includes the effects of lattice imperfections and in general is not equal to the ideal design orbit of the ring.

[3] The vectors $\vec{n}_1$ and $\vec{n}_2$ should not be confused with the quantization axis of the spin eigenstates, which is conventionally denoted by $\vec{n}$ by workers in the field, see, e.g. [2,13].