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Commentary

Analysis of an x-ray mirror made from piezoelectric bimorph

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A B S T R A C T

Theoretical analysis of the mechanical behavior of an x-ray mirror made from piezoelectric bimorph is presented. A complete two-dimensional relationship between the radius of curvature of the mirror and the applied voltage is derived. The accuracy of this relationship is studied by comparing the figures calculated by the relationship and Finite Element Analysis. The influences of several critical parameters in the relationship on the radius of curvature are analyzed. It is found that piezoelectric coefficient d_{31} is the main material property parameter that dominates the radius of curvature, and that the optimal thickness of PZT plate corresponding to largest bending range is 2.5 times of that of faceplate. It is demonstrated that the relationship is helpful for us to complete the primary design of the x-ray mirror made from piezoelectric bimorph.

1. Introduction

Piezoelectric Bimorph Mirrors (PBMs) (which are used in this paper to indicate x-ray mirrors made from piezoelectric bimorphs) are currently extensively used by the synchrotron radiation communities as x-ray focusing optics, as PBM can dynamically changes its curvature to be able to adjust its focusing properties to different beamline geometries or to the variations of the grazing angle [1]. With respect to mechanically bent mirrors, the PBM with a greater number of degrees of freedom allows the users to correct low or mid-spatial frequency errors, including polishing defects, deformations due to gravity or clamping, and photon-induced thermal bumps. Furthermore, PBM has the capability to create non-Gaussian focal spot profiles and correct the wavefront distortions induced by other optical elements on beamlines [2].

Adopted from astronomy and laser optics, PBM used as an x-ray focusing device was first applied at the European Synchrotron Radiation Facility (ESRF) [3]. The initial structure of PBM consists of two bonded zirconate lead litanate (PZT) piezoelectric ceramic plates and two super-polished faceplates glued onto the bimorph. Metallic conducting electrodes are deposited at all the three interfaces, and the top and bottom electrodes are used as ground electrodes while the central electrode is used as a control electrode. Fig. 1 in Ref. [3] shows the working principle of it: with two PZT plates being polarized normal to their reflecting surfaces, any voltage applied to the PBM will cause opposite changes of the lateral dimensions of the PZT plates: one plate shrinks while the other one expands, and therefore result in the bending of the PBM. Several PBMs based on this structure have been commercially designed and manufactured by Thales-SESO and used on many synchrotron beamlines. It has been demonstrated that such a PBM with super-polished surface could achieve sub-nanometer rms figure error with respect to a given ellipse by adaptive zone corrections [2]. PBM has a limited length (usually less than 200 mm) due to limitations in the fabricating of PZT, thus multi-segmented PBMs which includes several bimorphs assembled side-by-side has been designed, manufactured and used to achieve long mirrors (ranging from 100 mm to 1800 mm) [4,5]. However, distortions on the reflecting surface of the multi-segmented PBMs, with varying degrees of severity, have been detected due to mechanical junctions between two adjacent bimorphs [6]. In order to address this problem, a next-generation PBM in which four piezo ceramic strips are bonded to the side faces of a monolithic substrate to avoid the junctions being directly located below the reflecting region has been recently developed [7]. This PBM has obtained 0.5 μrad rms slope error for a range of ellipses, and no evident distortions induced by mechanical junctions have been found. This new design has been patented to Thales-SESO [8]. A similar design has been developed by Osaka University [9]. In this case, diffraction-limited focusing was realized and the shape accuracy nearly satisfied Rayleigh's quarter-wavelength criterion. The next-generation PBM bends less than an initial structure of PBM of comparable length and thickness. However, this bending limitation can be overcome by making a thinner substrate. In this paper, we only concentrate our study on the initial structure of PBM, and this analysis method can be extended to the next-generation PBM.

So far all the published papers on PBM just described the fabrication, characterization in metrology laboratory and commission

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on beamline of the PBM except the paper published by J. Susini et al. [10]. Using the elastic theory of bending beam, J. Susini et al. [10] deduced the relationship between radius of curvature of the PBM and applied voltage given as: $R=t^2/(\alpha d_{31}V)$, where R and t are the radius of curvature and thickness of the PBM respectively, d_{31} the piezoelectric coefficient of the PZT, V the applied voltage, and α a coupling factor depending upon the material properties and geometry of the PBM. α is a constant for a given PBM and calculated by Finite Element Analysis (FEA). The expression above has been generally used for the primary design of the PBM with the help of FEA. However, a complete theoretical relationship between the radius of curvature and applied voltage is absent nowadays.

In this paper, a systematic theoretical analysis of the electro-mechanical behavior of the PBM is presented. First, a complete relationship between radius of curvature and applied voltage is deduced. Second, the accuracy of the relationship is studied. Finally, the influences of several critical parameters on the radius of curvature are analyzed to provide a helpful guidance for the design of PBM.

2. Derivation of the theoretical relationship between radius of curvature and applied voltage

The geometry of the PBM under consideration is defined in Fig. 1. It is a laminated structure with rectangular reflecting surface of length L and width W, consisting of two PZT plates with thickness t_1 and two super-polished faceplates with thickness t_2. The origin of coordinate system is located at the center of the PBM. Both PZT plates are polarized along the z direction. The polarization vectors $\mathbf{P}_1$ and $\mathbf{P}_2$ of two plates are parallel to each other. Any voltage applied to the interface of the two PZT plates will lead to the bending of the PBM. We would like to know the relationship between the applied voltage and the generated radius of curvature.

The general equations that govern the electro-mechanical behavior of the PZT are thermodynamic equations of state [11], which include a variety of equations with different dependent and independent variables. In this derivation, we choose to use the following set of equations:

$$S = d'E + s^E T$$
$$D = dT + \varepsilon^T E \quad (1)$$

where $\mathbf{S}$ and $\mathbf{T}$ are strain and stress tensors respectively, $\mathbf{E}$ and $\mathbf{D}$ the electric field and electric displacement field respectively, $\mathbf{s^E}$ the elastic compliance matrix at constant electric field, $\boldsymbol{\varepsilon}^\mathbf{T}$ the dielectric permittivity matrix at constant stress, $\mathbf{d}$ the piezoelectric strain matrix of the PZT, and $\mathbf{d'}$ the transposed matrix of $\mathbf{d}$.

Due to the symmetry of poled PZT, independent components of its dielectric, elastic and piezoelectric matrixes are reduced and described as following: (i) The dielectric permittivity matrix is diagonal, with $\varepsilon_{11}^T = \varepsilon_{22}^T$. (ii) The elastic compliance matrix is symmetric and contains only five independent elements namely s_{11}^E, s_{12}^E, s_{13}^E, s_{33}^E and s_{44}^E, with $s_{11}^E = s_{22}^E$, $s_{23}^E = s_{13}^E$, $s_{44}^E = s_{55}^E$ and $s_{66}^E = 2(s_{11}^E - s_{12}^E)$. (iii) Most of the piezoelectric matrix elements are zero except d_{31}, d_{32}, d_{33}, d_{24} and d_{15}, with $d_{31} = d_{32}$ and $d_{24} = d_{15}$. Thus Eq. (1) can be written out in full as follows:

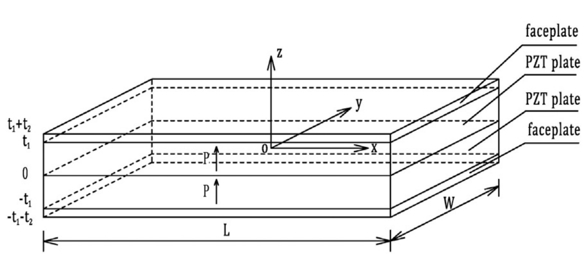

Fig. 1. PBM geometry and the relevant parameters.

$$S_1 = d_{31}E_3 + s_{11}^E T_1 + s_{12}^E T_2 + s_{13}^E T_3$$
$$S_2 = d_{31}E_3 + s_{12}^E T_1 + s_{11}^E T_2 + s_{13}^E T_3$$
$$S_3 = d_{33}E_3 + s_{13}^E T_1 + s_{13}^E T_2 + s_{33}^E T_3$$
$$S_4 = s_{44}^E T_4$$
$$S_5 = s_{44}^E T_5$$
$$S_6 = 2(s_{11}^E - s_{12}^E)T_6$$
$$D_1 = d_{15}T_5$$
$$D_2 = d_{24}T_4$$
$$D_3 = d_{31}T_1 + d_{31}T_2 + d_{33}T_3 + \varepsilon_{33}^T E_3 \quad (2)$$

Since the total thickness of the PBM is much smaller than its length L and width W, it can be treated by the use of elastic theory of bending beam. The PBM is then assumed to perform a pure bending. The other assumptions of the use of this theory are discussed by Roark [12]. Therefore, the normal strains: S_1 and S_2 can be expressed as: $S_1=z/r_1$ and $S_2=z/r_2$, where r_1 and r_2 are the radii of curvature in the x and y directions respectively, and the shear stresses: T_4, T_5 and T_6 can be ignored. Eq. (2) can be further simplified if the stress in the z direction, $T_3 \equiv 0$. This is justified here because there will be no external constraints on either the top or bottom surface of the PBM, leaving it completely free to strain in that direction. Thus Eq. (2) can be written in following format:

$$-z/r_1 = d_{31}E_3 + s_{11}^E T_1 + s_{12}^E T_2 \quad (3)$$

$$-z/r_2 = d_{31}E_3 + s_{12}^E T_1 + s_{11}^E T_2 \quad (4)$$

$$D_3 = d_{31}T_1 + d_{31}T_2 + \varepsilon_{33}^T E_3 \quad (5)$$

The third expression of Eq. (2) is neglected here because the strain in the z direction is without considering. Using Eqs. (3) and (4), T_1 and T_2 can be expressed as follows:

$$T_1 = [(s_{12}^E/r_2 - s_{11}^E/r_1)z - (s_{11}^E - s_{12}^E)dE]/[(s_{11}^E)^2 - (s_{12}^E)^2] \quad (6)$$

$$T_2 = [(s_{12}^E/r_1 - s_{11}^E/r_2)z - (s_{11}^E - s_{12}^E)dE]/[(s_{11}^E)^2 - (s_{12}^E)^2] \quad (7)$$

Substituting Eqs. (6) and (7) into Eq. (5) gives:

$$D_3 = [d_{31}(s_{12}^E/r_1 + s_{12}^E/r_2 - s_{11}^E/r_1 - s_{11}^E/r_2)z]/[(s_{11}^E)^2 - (s_{12}^E)^2] + [\varepsilon_{33}^T - 2d_{31}^2/(s_{11}^E + s_{12}^E)]E_3 \quad (8)$$

Since there is no free charge distribution across the thickness of the PZT plates, the derivative of D_3 with respect to z is zero:

$$dD_3/dz = 0 \quad (9)$$

The voltage applied at the interface between two PZT plates can be calculated by integrating the electric field:

$$V = \int_0^{t_1} E_3 dz \quad (10)$$

Eq. (10) is just applied to the top PZT plate. Therefore we will continue the derivations based on the top half of the PBM. Using Eq. (10) combined with Eq. (8) and Eq. (9) allows us to derive the electric field distribution along the z direction:

$$E = A(z - t_1/2) + V/t_1 \quad (11)$$

where A is a constant defined for convenience:

$$A = [d_{31}(s_{11}^E/r_1 + s_{11}^E/r_2 - s_{12}^E/r_1 - s_{12}^E/r_2)]/\{(s_{11}^E - s_{12}^E)[\varepsilon_{33}^T(s_{11}^E + s_{12}^E) - 2d_{31}^2]\} \quad (12)$$

Substituting Eq. (11) in Eqs. (6) and (7) gives:

$$T_1 = Bz + C \quad (13)$$

$$T_2 = Dz + C \quad (14)$$

where B, C and D are also the constants defined for convenience: