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Sum-fit method of analysis of nuclear decay spectra affected by extending dead-time

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ABSTRACT

We present a method of analysis of nuclear decay spectra affected by extending dead-time, in which a non-trivial, but relatively simple correction for dead-time losses is applied to produce an estimate of the spectrum in the absence of dead-time, with negligible bias and negligible deviation from the original Poissonian statistics. The method requires the measurement of arrival times of all observed decay events, but it does not require any knowledge about the dead-time or the nuclear decay. The parameters of nuclear decay can be determined in a subsequent straight-forward analysis of the estimated dead-time free spectra, individually or in combinations. In a typical realistic situation this approach is expected to contribute 0.001% or less to the systematic error of nuclear half-life or activity. Compared to the exact method of analysis presented in our earlier publication, the present method is much easier to implement and produces the results significantly faster.

1. Introduction

Dead-time of the apparatus used for particle detection affects every measurement of a nuclear half-life, yet its nature and extent are not exactly known. Consequently, the data obtained in such measurements yield results that depend on some educated estimates of the dead-time effects. These effects increase with the product of the dead-time per event and the event rate that would be observed in the absence of dead-time (*i.e.*, the ideal rate), so the uncertainty in the estimates increasingly limit the accuracy of the measured half-lives. Therefore, there is a need for a proven standard method of measurement and analysis that would provide the best accuracy and precision of the results for a given number of measured events.

Such a method has been recently proposed [1], extended [2], and reviewed [3]. It requires a measurement of the time of each particle-detection event, which can easily be accomplished using a suitable time-to-digital converter (TDC) [1]. The key element in the analysis is the imposition of a known, sufficiently large extending dead-time to the measured (*i.e.*, observed) sequence of events in order to produce a set of surviving events for which the effects of the actual dead-time are completely obliterated by the effects of the imposed dead-time. As a result, the dead-time following each surviving event and the live time preceding each surviving event are known exactly, which provides for an exact statistical analysis of the time intervals between consecutive surviving events. This method was validated using simulated data for the beta-decay of ^{26m}Al (half-life $T_{1/2}$ = 6.3452 s), with an assumed constant background event-rate of $B = 1$ s^{-1} and an initial true decay rate A ranging from 100 s^{-1} to 100,000 s^{-1}. The imposed extending dead-time per event τ ranged from 2 μs to 512 μs, so that $A\tau$ ranged from 2×10^{-4} to as high as 51.2.

However, it turns out that the time it takes to analyze the data using this method for the given total number of events increases drastically as the event rate decreases. This is mainly because the number of events per sample decreases, so an increasingly larger number of samples must be analyzed. While effective analysis of a large number of samples is only a matter of available resources (such as processor speed and memory size), an additional problem may present itself if the number of events per sample is too small: *viz.*, the distributions of the maximum-likelihood values of the fitting-function parameters obtained in the analysis of each sample may broaden so much that they become affected by the physical restrictions of the problem, such as the requirement that all parameters have positive values. This leads to an increased systematic error and a biased result. Moreover, if the affected samples are systematically ignored, this has the same effect.

To avoid this problem and to speed-up the analysis considerably, a "sum-fit" approach similar to that described in Ref. [4] could be used, assuming that it can be modified for use with decay spectra affected by *extending* dead-time. The data analysis would then involve (a) producing a decay spectrum (*i.e.*, a time histogram) of the surviving events for each

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sample, (b) correcting the number of events in each channel of each decay spectrum for dead-time losses, and, as an option, (c) grouping and combining channel-wise the resulting dead-time-corrected (dead-time-free) decay spectra into a single decay spectrum (per group) for an analysis to determine the nuclear half-life. While parts (a) and (c) of this procedure are straightforward, the best algorithm for part (b) has yet to be determined.

In this paper we evaluate two methods in terms of bias, and then determine the corrections needed to reduce that bias, so that the methods can be applied in high-precision work in typical realistic situations, within a specified broad range of parameters that define the problem. Finally, we show that one of the methods of analysis is better than the other in every respect and show that (with the applied correction for bias) it produces an estimate of the decay spectrum in the absence of dead-time with negligible deviation from the original Poissonian statistics.

2. The methods

For an infinitely long sequence of Poissonian events that starts at a known time and under known circumstances, the correction for the effects of an imposed dead-time depends only on the nature of the dead-time and the product of the ideal rate ρ of the events and the imposed dead-time per event τ. This product is related to the surviving event rate R as [5]

$$R\tau = \rho\tau/(1+\rho\tau) \tag{1}$$

for non-extending dead-time and

$$R\tau = \rho\tau \exp(-\rho\tau) \tag{2}$$

for extending dead-time.

In practice, ρ is not known *a priori* and is usually derived from the measurement of R. If ρ is constant (which will be assumed from this point on), then R is constant as well and can be measured by counting the surviving events during a chosen time interval Δt, then dividing the result (n_{sur}) by Δt. If the nature of the dead-time and its value τ are known as well, then Eq. (1) or Eq. (2), whichever applies, can be used to derive a measure of ρ based on the measured value of R. The number n_{o} of original Poissonian events (*i.e.*, those that would be observed in the absence of dead-time) then can be estimated as $\rho\Delta t$ and this quantity (n_{c}), which generally is a non-integer, can be referred to as the *(dead-time) corrected* number of events.

Although it may be hoped that the distribution of n_{c} values obtained this way (or using some other appropriate method) is in agreement with the Poissonian distribution of the n_{o} values, or that at least the two distributions have the same total, mean and variance for any chosen value of Δt, neither of these possibilities is generally true, except in the trivial case of $\tau = 0$. Moreover, quantified deviations from these ideal properties may depend on the way the dead-time correction is applied and generally may depend on two characteristic dimensionless parameters, such as $\rho\tau$ and $\tau/\Delta t$, formed out of the three characteristic parameters, which have dimensions.

Here we evaluate the dead-time correction procedures based on two different methods. Specifically, the first method is intuitive and straightforward, and may have been in general use already. The second method is non-trivial and it is proposed here because of its better accuracy and better stability compared to the first method. We also show how each method can be corrected in order to reduce its inherent bias to a level acceptable in typical precision measurements of nuclear half-lives.

2.1. Method 1

We first evaluate the dead-time correction procedure based on the expression

$$n_{\mathrm{c}} = n_{\mathrm{sur}} \times \Delta t/t_{\mathrm{live}}, \tag{3}$$

where t_{live} is the total live time within the time interval (or channel width) Δt. This relation is simple and intuitive, since either $n_{\mathrm{c}}/\Delta t$ or $n_{\mathrm{sur}}/t_{\mathrm{live}}$ can be used as estimates of ρ, regardless of the nature of dead-time, and so Eq. (3) applies to both extending and non-extending dead-time cases.

For the case of non-extending dead-time, Eq. (3) is consistent with Eq. (6) of Ref. [4] if $t_{\mathrm{live}} = \Delta t - n_{\mathrm{sur}}\tau$, which is statistically correct, although the actual values of t_{live} may slightly vary in successive channels as the dead-time following the last event in the current channel may randomly extend into the next channel and the dead-time of the last event in the previous channel may randomly extend into the current channel. Provided that the arrival times of all observed events are recorded and the imposed non-extending dead-time is much larger than the actual dead-time in its absence, it is possible to correct for the majority of this effect. It was found that this correction improves the accuracy of analysis of the decay spectra. However, further discussion of this case is outside of the scope of this paper.

For the case of extending dead-time (and constant ρ), Eq. (3) is consistent with the exact method of analysis described in Ref. [1] when it is applied to an individual channel. Specifically, this means replacing t_{f} in Eqs. (14) and (16) of Ref. [1] with Δt.

If arrival times of all events observed in a measurement are recorded, then the imposition of a long-enough extending dead-time to the sequence of observed events eventually produces a sequence of surviving events that is the same as if an equal extending dead-time was imposed on the original sequence of Poissonian events [1]. Consequently, the live time preceding each surviving event as well as the dead-time following it are known exactly [1]. Therefore, after binning, t_{live} can be determined exactly for any channel of a time histogram, and Eq. (3) can be applied in a straight-forward way.

However, the correction for dead-time based on Eq. (3) may run into problems under some circumstances. Specifically, this method (a) does not change the contents of a channel without any surviving events, regardless of the channel's live-time fraction and regardless of the original number of events present, (b) overestimates the number of events in channels that have one surviving event and had one event originally, (c) cannot be applied to channels in which live time is zero, and (d) may lead to a serious overcorrection for channels in which live time is much smaller than the channel duration. Consequently, this method is not expected to be useful when $\rho\tau$ and $\tau/\Delta t$ are large enough to result in time histograms with a significant number of channels having the problematic properties listed above.

Since the mathematical complexities associated with the effect of extending dead-time on time-binned Poissonian events are overwhelming [5], we choose to study the effects of dead-time correction based on Eq. (3) empirically and heuristically, using simulated events. To that effect, a Monte Carlo procedure was used to produce ten Poissonian sequences of events, corresponding to different random number generator seeds. Each sequence contained approximately 60 million events, which is a typical number of events to be collected in an actual precision measurement of a nuclear half-life. More than one set was produced in the simulation in order to be able to ascertain the magnitude of statistical variations at the level of 60 million event statistics.

The ideal rate used in the simulations was $\rho = 10$ events/s, which was chosen arbitrarily, keeping in mind that the scaling parameters are $\rho\tau$ and $\tau/\Delta t$, so that the same results are obtained for x times this rate, as long as τ and Δt are reduced by the same factor x at the same time. This is equivalent to choosing an x times longer unit of time.

The imposed dead-time parameter τ ranged from 100 ns to 5 ms ($\rho\tau$ ranging from 10^{-6} to 0.05), while $\tau/\Delta t$ ranged from 10^{-8} to 0.002, with three values per decade for each parameter. All combinations of the selected τ and $\tau/\Delta t$ values were used in the analysis. The corresponding values of Δt ranged from 50 μs to $5 \cdot 10^5$ s.

Note that in this analysis the ideal rate and the original number of Poissonian events in each channel are known exactly. However, these values were not used in the analysis. They were used only to determine the bias of the results.