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An improved method of lifetime measurement of nuclei in radioactive decay chain

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ABSTRACT

We present an improved statistical method for the calculation of mean lifetime of nuclei in a decay chain with an uncertain relation between mother and daughter nuclei. The method is based on the formation of time distribution of intervals between mother and daughter nuclei, without trying to set the exact mother-daughter nuclei relationship. If there is a coincidence of mother and daughter nuclei decays, the sum of these distributions has flat term on which an exponential term is superimposed. Parameters of this exponential function allow lifetime of daughter nucleus to be extracted. The method is tested on Monte Carlo simulation data.

1. Introduction

Basic stochastic methods are intensely used in today's experiments that aim to determine lifetime of unstable nuclei. Most of the already developed methods for calculating mean lifetime of daughter element in the decay chain strongly depend on correct identification of the exact mother-daughter pair in the time sequence data. In case of low activity of the mother isotope (compared to decay constant λ of the daughter) it is easy to determine the mother-daughter pair with high probability, but the statistics is low. However, raising the activity will raise the statistics, but decrease the probability for correct pairing of mother-daughter nuclei.

Bernas et al. (Ref. [1]) analyzed the time correlation between the detection times of a fragment of interest and of a subsequent β particle. In order to obtain the beta decay half-life they formed the distribution of time intervals only between the first β detected after each fragment. They also provided the random distribution by collecting time intervals between the last β occurring before a fragment and the fragment. In this article, we present a technique for determination of mean lifetime of nuclei in decay chain without the need to know the exact relationship between particular decay of the parent and daughter nuclei. This stochastic method is explained in Section 2. One of the benefits of this approach is that the increase of activity of mother nuclei is followed by decrease of error of the determined lifetime. Moreover, the method is not activity dependent, meaning that activity may vary during the data acquisition, which may be the case in many real situations. We also pay special attention to determination of errors depending on the activity, for fixed time of measurement and constant activity. The presented concept is checked using an extensive set of Monte Carlo simulation data. The simulation program is custom made and developed by our group. The results of the MC simulation are shown in Section 4 and Section 5.

2. Statistical procedure of formation of time intervals distribution

Let us consider the following decay chain $X \rightarrow Y \rightarrow Z$. We introduce two parameters that are of importance for our analysis: activity $A(X)$ of the mother nuclide X, and the decay constant $\lambda(Y)$ of the daughter nuclide Y. Our aim is to determine the mean lifetime of the daughter nuclide, which is the inverse of the decay constant, $\tau=1/\lambda$. If the radioactive equilibrium is achieved, meaning $A(X)=A(Y)$, we distinguish three cases:

1) If the mean lifetime of the Y nuclide is much longer than the time between two X decays, meaning $A(X) > > \lambda(Y)$, there is a strong possibility that successive decays of X and Y nuclei do not belong to the same $X \rightarrow Y \rightarrow Z$ chain.
2) Conversely, if the mean lifetime of the Y nuclide is much shorter than the time between two X decays, meaning $A(X) \ll \lambda(Y)$, there is strong possibility that the decay of a nucleus X is followed by the decay of nucleus Y, which is the daughter nucleus of the mentioned nucleus X.
3) If $A(X) \sim \lambda$, it is not clear what the relationship between decay of X and the following decay of Y is.

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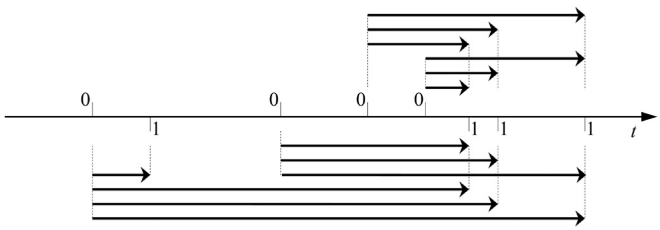

Fig. 1. A time sequence with the applied procedure of collecting the time intervals between succeeding start-stop pairs of signals. The start and the stop signals are labeled by 0's (X decay events) and 1's (Y decay events), respectively. The arrows correspond to the collected time intervals.

As far as the second case is concerned, the identification of the right decay is good. It is possible to form a set of time differences between the decays of the mother nuclei and the decays of the corresponding daughter nuclei. The average of these differences is the estimate of the mean lifetime of nucleus Y. The downside of this case is that the statistics is low since the activity of nucleus X is low.

We are interested in the first case where the statistics is high but the identification of the right decay is poor. Especially if there is an abundance of nuclei X around the detector, there is a possibility that after the formation of one Y nucleus one or more other Y nuclei may be detected before the first one decay. Because of the simultaneous presence of many Y nuclei it is not possible to determine the order in which the nuclei decay, thus it is not possible to determine the lifetimes of those nuclei (Fig. 1).

We observe an array of signals originating from the decay of X and Y nuclei that are randomly settled on time scale. Start signals arise at the time instants of decay of X nuclei (X event). Also, stop signals arise at instants of decay of Y nuclei (Y event). This array of start and stop signals may be treated in many different ways in order to obtain the decay constant of the daughter radionuclide. One possibility is to join weights for the probability of correct pairing with each pair of start-stop signals formed. In other words, it is possible to calculate the probability for each pair that it is a real coincident pair, as in Ref. [2]. Another solution is to neglect all the cases where a start signal is not followed by a stop signal, but by another start signal; and to keep only sequences with clear start-stop coincident pairs, as it is done in Ref. [3]. Both methods have up and downsides.

Our approach is not to neglect any signal in order to keep high statistics. We pair different start and stop signals, but we do not assign probability of correct pairing for each pair. Instead we build and investigate the time distribution of intervals between a start signal and all the following stop signals, as shown in Fig. 1.

Let us denote the following variables:

A – the activity of the parent X, ε_X, ε_Y – the efficiency for detection start signal (decay of X) and stop signal (decay of Y),

$p_{n(X\to Y)}(t)$ – the probability that in the interval [0, t] exactly n stop signals which originate only from random coincidence between X and Y decay occur,

$p_{n(X\to YC)}(t)$ – the probability that in the interval [0, t] exactly n stop signals occur, in which one true coincidence between X and Y decay is found (there can be only one!),

$p_n(t)$– the probability that in the interval [0, t] exactly n stop signals occur regardless of the origin,

$p_n^{Coll}(t)$ - the probability that in the interval [0, t] exactly n time intervals are collected - which include detector efficiency ε_X for detection start signal (decay of X) and detector efficiency ε_Y for detection stop signal (decay of Y).

It is clear that:

$$p_n = p_{n(X\to Y)} + p_{n(X\to YC)}$$

The probability to collect n uncorrelated time intervals are given by recursive formula:

$$dp^{Coll}_{(n+1)(X\to Y)}(t) = \varepsilon_X p_{n(X\to Y)}(t')e^{-A(t-t')}e^{-\lambda(t-t')}\varepsilon_Y A dt'$$

$$p^{Coll}_{(n+1)(X\to Y)}(t) = \int_0^t \varepsilon_X p_{n(X\to Y)}(t')e^{-A(t-t')}e^{-\lambda(t-t')}\varepsilon_Y A dt'$$

$$p^{Coll}_{n(X\to Y)}(t) = \varepsilon_X\varepsilon_Y p_{n(X\to Y)}(t),$$

where

$$p_{n(X\to Y)}(t) = A\frac{(At)^{n-1}}{(n-1)!}e^{-(A+\lambda)t}$$

The probability to collect n time intervals with one true coincidence:

$$dp^{Coll}_{(n+1)(X\to YC)}(t) = \varepsilon_X p_{n(X\to Y)}(t')e^{-A(t-t')}e^{-\lambda(t-t')}\varepsilon_Y \lambda dt' + \varepsilon_X p_{n(X\to YC)}(t')e^{-A(t-t')}\varepsilon_Y A dt'$$

$$p^{Coll}_{(n+1)(X\to YC)}(t) = \varepsilon_X\varepsilon_Y\int_0^t e^{-A(t-t')}(\lambda p_{n(X\to Y)}(t')e^{-\lambda(t-t')} + A p_{n(X\to YC)}(t'))dt'$$

$$p^{Coll}_{n(X\to YC)}(t) = \varepsilon_X\varepsilon_Y p_{n(X\to YC)}(t)$$

where

$$p_{n(X\to YC)}(t) = A\frac{(n-1)(At)^{n-2}}{(n-1)!}e^{-(A+\lambda)t}(e^{\lambda t}-1) + \lambda\frac{(At)^{n-1}}{(n-1)!}e^{-(A+\lambda)t}$$

The sum of probability distributions for all possible n (from 1 to infinity) is:

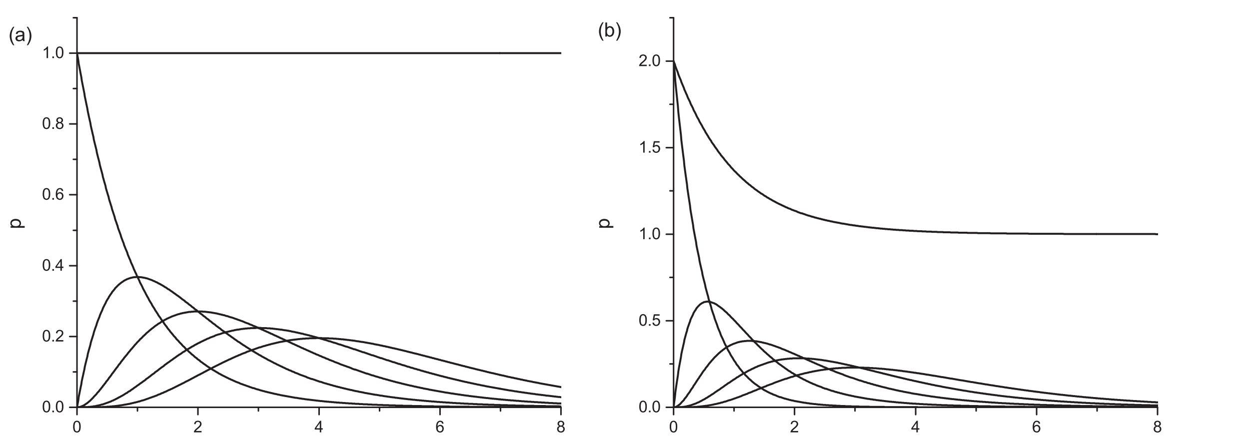

Fig. 2. Probabilities graphics for the first 5 intervals between start and stop signals and sums of probabilities for all intervals without (a) and with (b) coincidence between start and stop signals.