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Optimal dimension of a layered hydrogen target for muon conversion and muonic x-ray spectroscopy

F. Mohamadsalehi[a], R. Gheisari[a,*], A. Avazpour[b]

[a] Physics Department, Persian Gulf University, Bushehr 75169, Iran
[b] Department of Physics, Yasouj University, Yasouj 75919, Iran

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ABSTRACT

In spectroscopic studies of nuclei (muonic x-ray spectroscopy and muon conversion), layered solid hydrogen targets are of key importance. The paper develops a mathematical model to optimize the dimension of solid deuterium (degrader) layer in a layered hydrogen structure of $H_2/T_2 + sD_2$ to prepare muon for the creation of muonic atoms. Muonic object balance equations have been constructed and solved. All muonic molecular resonances have been taken into account. The results show good accordance with muon experiments.

1. Introduction

During the past seventeen years, the cold hydrogen film method has played an important role to extend muonic atom spectroscopy and muon conversion [1–13]. In recent years, two layered solid hydrogen targets were considered theoretically to model muon conversion [7], x-ray yield [9,11–13], time delay of muonic atom [11], muonic atom loss [12] and effect of the presence of protons on x-ray production [13]. In September 2016, the 6th Yamada workshop on muonic x and gamma-ray spectroscopy was held in Osaka University. The main goal of this workshop was to open up new opportunities for researchers from academia and industry to use muonic x-rays and gamma-rays, which are emitted from material after a muon stopped in it. In October 2016 year, workshops focusing on high resolution muonic x-ray spectroscopy of medium and high-Z elements were held in Paul Scherrer Institute (PSI). Muonic x-rays have never been studied still with highly efficient detector arrays covering all solid angles. In these spectroscopic studies, the geometry of solid targets is of key importance. In this paper we have the goal to optimize the dimension of the solid deuterium (sD_2) in the target of $H_2/T_2 + sD_2$ (frozen on a thin gold foil kept at $\sim 3\,K$) to prepare muon for the creation of μt_{1s} muonic atoms. In Section 2, we develop a mathematical model that considers the geometry of media and takes into account all the $dt\mu$ muonic molecular resonances, muonic atom escape and time delay of muonic atom. The results are given in Section 3. In Section 4, we summarize the paper and give the future perspectives of this study as conclusion.

2. Materials and method

A layer of H_2/T_2 is usually used to convert the stopped muons into a μt_{1s} beam [7,14,15]. In this respect, $Y_\mu \simeq 32\%$ of injected muons are stopped in a layer of H_2/T_2 with a thickness of 3.4 mg/cm^2 [14]. The forward emission yield ($Y_{\mu t}$) equals approximately 25% of the stopped muons. The sD_2 layer plays two main roles (μt_{1s} degrader and moderator). μt_{1s} atoms make elastic scattering on deuterium nuclei, until fall down to reach the resonance regions [16,17]. A considerable fraction of μt_{1s} muonic atoms can then be absorbed to form μdt molecules [10] (see Fig. 1). These muonic molecules fuse fast on the $\sim ps$ time scale (see Table 1). The muon can be released with a high probability of $(1-\omega_s^{eff.})$, where $\omega_s^{eff.}$ is the effective muon sticking coefficient [18] in the μdt branch. In solid deuterium media, the free muon can form a μd_{1s} muonic atom. μd_{1s} muonic atom occurs μdd molecular formation. The deuteron nuclei in this molecule fuse on the $\simeq 2ns$ time scale (see Table 1) and the muon repeats the same processes with a significant probability of $(1-\omega_{dd})$ before its decay. ω_{dd} is the effective muon sticking coefficient [19] in the μdd branch. In Fig. 2 a comprehensive muonic network is given. The corresponding set of muonic object equations is written in the following.

$$\frac{\partial N_\mu(z,t)}{\partial t} = -(\lambda_0 + \lambda_a \phi) N_\mu(z,t) + \lambda^f_{\mu dt} N_{\mu dt}(z,t)(1-\omega_s^{eff.}) + \lambda^f_{\mu dd} N_{\mu dd}(z,t)(1-0.58\omega_{dd}), \tag{1}$$

* Corresponding author.
E-mail address: gheisari@pgu.ac.ir (R. Gheisari).

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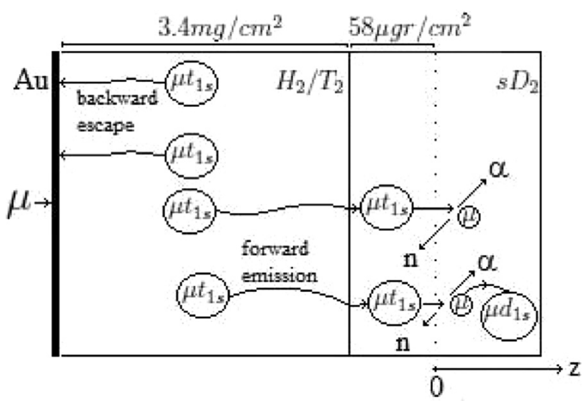

Fig. 1. A layered hydrogen structure of $H_2/T_2 + sD_2$. The layer of H_2/T_2 with a thickness of 3.4 mg/cm^2 converts the stopped muons into a μt_{1s} muonic atom beam. The sD_2 layer plays two main roles (μt_{1s} atom degrader and moderator). The thickness of the degrader is ≃58 μgr/cm^2.

$$\frac{\partial N_{\mu t_{1s}^{E=2.2}}(z,t)}{\partial t} = -\overline{\cos(\theta_{\mu t})}v_{\mu t}(2.2)\frac{\partial N_{\mu t_{1s}^{E=2.2}}(z,t)}{\partial z} - \phi\lambda_{\mu dt}^{non.}N_{\mu t_{1s}^{E=2.2}}(z,t) - (\lambda_0 + \Sigma_s(2.2)v_{\mu t}(2.2))N_{\mu t_{1s}^{E=2.2}}(z,t), \quad (2)$$

$$\frac{\partial N_{\mu t_{1s}^{E=1.5}}(z,t)}{\partial t} = -\overline{\cos(\theta_{\mu t})}v_{\mu t}(1.5)\frac{\partial N_{\mu t_{1s}^{E=1.5}}(z,t)}{\partial z} - \phi\lambda_{\mu dt}^{res.=1.5}N_{\mu t_{1s}^{E=1.5}}(z,t) - (\lambda_0 + \Sigma_s(1.5)v_{\mu t}(1.5))N_{\mu t_{1s}^{E=1.5}}(z,t) + \int_{1.5}^{2.2}\frac{\Sigma_s(E)N_{\mu t_{1s}^{E}}(z,t)v_{\mu t}(E)dE}{E(1-\alpha)}, \quad (3)$$

$$\frac{\partial N_{\mu t_{1s}^{E=1}}(z,t)}{\partial t} = -\overline{\cos(\theta_{\mu t})}v_{\mu t}(1)\frac{\partial N_{\mu t_{1s}^{E=1}}(z,t)}{\partial z} - \phi\lambda_{\mu dt}^{res.=1}N_{\mu t_{1s}^{E=1}}(z,t) - (\lambda_0 + \Sigma_s(1)v_{\mu t}(1))N_{\mu t_{1s}^{E=1}}(z,t) + \int_{1}^{2.2}\frac{\Sigma_s(E)N_{\mu t_{1s}^{E}}(z,t)v_{\mu t}(E)dE}{E(1-\alpha)}, \quad (4)$$

$$\frac{\partial N_{\mu t_{1s}^{E=0.47}}(z,t)}{\partial t} = -\overline{\cos(\theta_{\mu t})}v_{\mu t}(0.47)\frac{\partial N_{\mu t_{1s}^{E=0.47}}(z,t)}{\partial z} - \phi\lambda_{\mu dt}^{res.=0.47}N_{\mu t_{1s}^{E=0.47}}(z,t) - (\lambda_0 + \Sigma_s(0.47)v_{\mu t}(0.47))N_{\mu t_{1s}^{E=0.47}}(z,t) + \int_{0.47}^{2.2}\frac{\Sigma_s(E)N_{\mu t_{1s}^{E}}(z,t)v_{\mu t}(E)dE}{E(1-\alpha)}, \quad (5)$$

$$\frac{\partial N_{\mu d_{1s}}(z,t)}{\partial t} = -\lambda_0 N_{\mu d_{1s}}(z,t) - \lambda_{\mu dd}^{3/2}\phi N_{\mu d_{1s}}(z,t) + \lambda_a\phi N_\mu(z,t), \quad (6)$$

Table 1
Required parameters for construction of muonic object balance equations.

$\lambda_{\mu dt}^{res.=0.47}$	Resonant formation rate of μdt for $E \simeq 0.47\ eV$	≃0.87 × 10^{10}	[16,17]
$\lambda_{\mu dt}^{res.=1}$	Resonant formation rate of μdt for $E = 1\ eV$	≃0.27 × 10^{10}	[16,17]
$\lambda_{\mu dt}^{res.=1.5}$	Resonant formation rate of μdt for $E = 1.5\ eV$	≃0.146 × 10^{10}	[16,17]
$\omega_s^{eff.}$	Muon sticking in μdt branch	≃0.005	[18]
ω_{dd}	Muon sticking in μdd branch	≃0.132	[19]
$\lambda_{\mu dd}^{f}$	μdd fusion rate	0.43 × 10^9	[19]
λ_a	Muonic atom formation rate	4 × 10^{12}	[22]
$\lambda_{\mu dt}^{f}$	μdt fusion rate	1.1 × 10^{12}	[22]
λ_0	Muonic decay rate	0.45585 × 10^6	[24]
$\lambda_{\mu dd}^{3/2}$	μdd formation rate from 3/2 spin state of μd_{1s}	2.87 × 10^6	[25]
$\lambda_{\mu dt}^{non.}$	Non-resonant formation rate of μdt	≈3 × 10^8	[26]

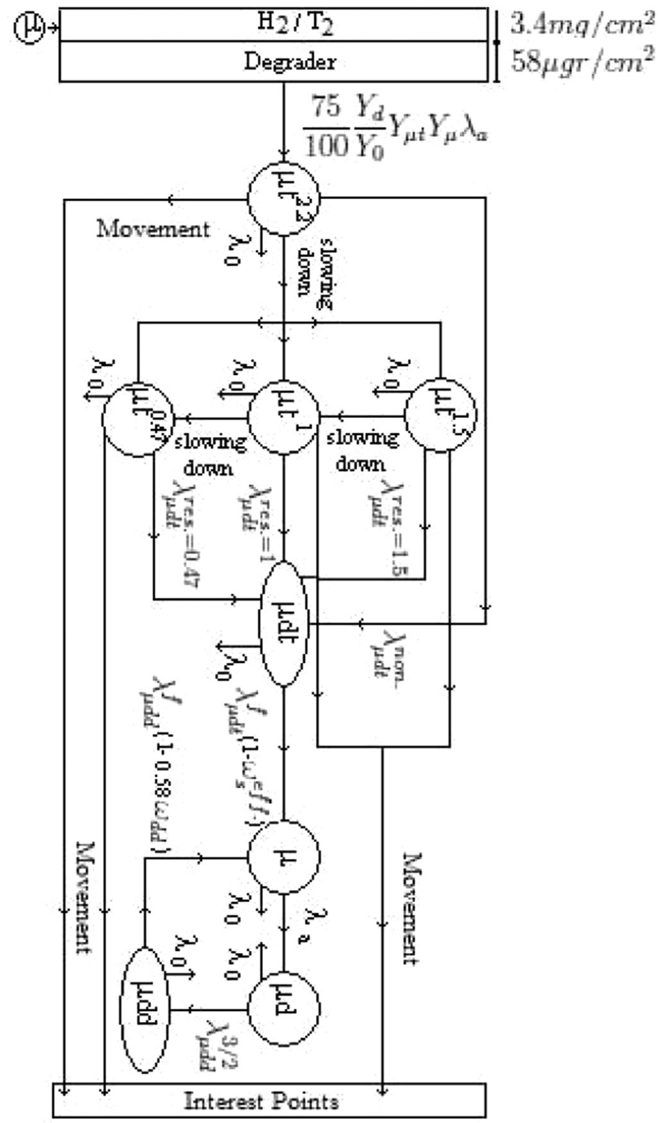

Fig. 2. Comprehensive muonic network in a layered hydrogen structure of $H_2/T_2 + sD_2$.

$$\frac{\partial N_{\mu dt}(z,t)}{\partial t} = \lambda_{\mu dt}^{res.=0.47}\phi N_{\mu t_{1s}^{E=0.47}}(z,t) + \lambda_{\mu dt}^{res.=1}\phi N_{\mu t_{1s}^{E=1}}(z,t) + \lambda_{\mu dt}^{res.=1.5}\phi N_{\mu t_{1s}^{E=1.5}}(z,t) + \lambda_{\mu dt}^{non.}\phi N_{\mu t_{1s}^{E=2.2}}(z,t) - (\lambda_{dt\mu}^{f} + \lambda_0)N_{\mu dt}(z,t), \quad (7)$$

$$\frac{\partial N_{\mu dd}(z,t)}{\partial t} = \phi\lambda_{\mu dd}^{3/2}N_{\mu d_{1s}}(z,t) - (\lambda_{\mu dd}^{f} + \lambda_0)N_{\mu dd}(z,t), \quad (8)$$

$$\frac{\partial X_c(z,t)}{\partial t} = \lambda_{\mu dd}^{f}N_{\mu dd}(z,t) + \lambda_{\mu dt}^{f}N_{\mu dt}(z,t). \quad (9)$$

Here, λ_0 is the muon decay rate, λ_a is the muonic atom formation rate, and $\lambda_{\mu dx}^{f}(x = d, t)$ is the fusion rate of the μdx molecule. $\overline{\cos(\theta_{\mu t})}$ denotes the average cosine of the μt_{1s} scattering angle (relative to z axis in the laboratory system). $v_{\mu t}(E)$ shows the μt_{1s} speed. E is the μt_{1s} kinetic energy expressed in eV units. α is the collision parameter. $\Sigma_s(E)$ shows the macroscopic scattering cross-section of μt_{1s} atom. $\lambda_{\mu dt}^{res.=E}$ is the μdt resonance formation rate for E, and $\lambda_{\mu dt}^{non.}$ being the μdt non-resonance formation rate. $\lambda_{\mu dd}^{3/2}$ is the μdd formation rate from $\frac{3}{2}$ spin state of μd_{1s} atom. ϕ is the density of the sD_2 expressed in LHD units ($1LHD = 4.25 \times 10^{22}cm^{-3}$ shows the liquid hydrogen density). $N_0^{O}(z,t)$ is the time evolution of the population of the muon or muonic atoms or