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Exploration and extension of an improved Riemann track fitting algorithm

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ABSTRACT

Recently, a new Riemann track fit which operates on translated and scaled measurements has been proposed. This study shows that the new Riemann fit is virtually as precise as popular approaches such as the Kalman filter or an iterative non-linear track fitting procedure, and significantly more precise than other, non-iterative circular track fitting approaches over a large range of measurement uncertainties. The fit is then extended in two directions: first, the measurements are allowed to lie on plane sensors of arbitrary orientation; second, the full error propagation from the measurements to the estimated circle parameters is computed. The covariance matrix of the estimated track parameters can therefore be computed without recourse to asymptotic properties, and is consequently valid for any number of observation. It does, however, assume normally distributed measurement errors. The calculations are validated on a simulated track sample and show excellent agreement with the theoretical expectations.

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1. Introduction

The trajectory of a charged particle through a high-energy physics detector system is governed by the equations of motion, given by the Lorentz force. In the general case with the presence of an inhomogeneous magnetic field, no analytical solutions to these equations exist, and one has to resort to numerical approaches such as a Runge–Kutta method of some order. In the very simplest case of a vanishing magnetic field, the track model is a straight line. Despite the intrinsic attractiveness of this simple track model, important properties as for instance the momentum and sign of charge of the particle cannot be estimated. The only case enabling such properties to be estimated and at the same time offering an analytical track model is a homogeneous magnetic field with field lines parallel to the beam direction. In this situation, the track model is a helix, or, in the bending plane of the particles, a circle.

Most inner tracking detector systems are therefore embedded in a nearly homogeneous magnetic field. Although general methods such as the Kalman filter [1] or global least-squares estimation [2] can be used in this case, track fitting in the bending plane can also be performed by simple, fast and non-iterative circle fitting methods such as the conformal mapping approach [3], the Karimäki method [4] or the Riemann fit [5]. These non-iterative methods are all based on some kind of simplifying approximation, which in general makes them less precise than more rigorous approaches.

In this paper, we present a thorough study of the precision of a recently proposed, improved Riemann track fit [6]. As suggested by Chernov [7], measurements are transformed in order to achieve invariance under translations and similarity transforms. We show that the improved Riemann fit is significantly more precise than some of the most popular, non-iterative approaches and virtually as precise as the Kalman filter, a global least-squares approach and an iterative, non-linear method.

In addition to estimating the track parameters, a track fitting algorithm should be able to assess the degree of uncertainty of these estimates. These uncertainties and the correlations between them are summarized in the covariance matrix. In [8], the covariance matrix of the track parameters was based on large-sample (asymptotic) properties of the sample covariance matrix of the observations. Here we present the full sequence of error propagation steps from the observations to the final track parameters. It is valid for any number of observations under the assumption of normally distributed measurement errors. The derivation is simpler in the statistically equivalent implementation of the Riemann fit proposed in [9], where the measurements are projected to the paraboloid $z = x^2 + y^2$ rather than to the Riemann sphere.

The paper is organized as follows. After a recollection of the basic concepts of the Riemann track fitting method, the recently introduced improvements to the original algorithm are reviewed. In a simulation

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study of a generic inner tracking system we show results comparing the precision of the improved Riemann fit with a set of circular track fitting methods. The derivation of the error propagation from the measurements to the estimated circle parameters is then presented and validated with simulated tracks. The paper is concluded by a summary and an outlook to further work.

2. The improved track fit on the Riemann paraboloid

The Riemann paraboloid is positioned on top of the (x, y)-plane with its global minimum at the origin of the plane. We assume that the measured points in the (x, y)-plane are given in Cartesian coordinates, $(u_i, v_i),\ i = 1, \dots, N$. A covariance matrix $\boldsymbol{V}_i$ is attached to each point. The covariance matrix is arbitrary in principle, but is required to be positive definite in order to avoid problems with rank-deficient matrices during the error propagation.

There are two important special cases. If the radial error of the point (u_i, v_i) can be neglected, its covariance matrix has the form:

$$\boldsymbol{V}_i = \frac{1}{\sqrt{u_i^2 + v_i^2}} \begin{pmatrix} \delta_i^2 u_i^2 + \sigma_i^2 v_i^2 & u_i v_i(\delta_i^2 - \sigma_i^2) \\ u_i v_i(\delta_i^2 - \sigma_i^2) & \delta_i^2 v_i^2 + \sigma_i^2 u_i^2 \end{pmatrix},$$

where σ_i is the standard deviation of the position error in the tangential direction, and δ_i is positive, but much smaller than σ_i, for instance $\delta_i = 0.01 \cdot \sigma_i$.

If the point (u_i, v_i) is a position measurement on a thin plane sensor with normal unit vector $\boldsymbol{a}_i = (a_{i,u}, a_{i,v})^{\mathsf{T}}$, its covariance matrix has the form:

$$\boldsymbol{V}_i = \begin{pmatrix} \delta_i^2 a_{i,u}^2 + \sigma_i^2 a_{i,v}^2 & a_{i,u}a_{i,v}(\delta_i^2 - \sigma_i^2) \\ a_{i,u}a_{i,v}(\delta_i^2 - \sigma_i^2) & \delta_i^2 a_{i,v}^2 + \sigma_i^2 a_{i,u}^2 \end{pmatrix},$$

where σ_i is the standard deviation of the position error of the sensor, and δ_i is again positive, but much smaller than σ_i, for instance $\delta_i = 0.01 \cdot \sigma_i$.

The mapping from the (u, v)-plane to the Riemann paraboloid is given by:

$$x_i = u_i$$
$$y_i = v_i$$
$$z_i = u_i^2 + v_i^2.$$

By this mapping, the circle in the plane with the equation

$$(u - u_0)^2 + (v - v_0)^2 = \rho^2$$

is mapped to the plane in 3D space with the equation

$$z - 2xu_0 - 2yv_0 = \rho^2 - u_0^2 - v_0^2.$$

A point with position $\boldsymbol{r} = (x, y, z)^{\mathsf{T}}$ satisfying $\boldsymbol{n}^{\mathsf{T}}\boldsymbol{r} + c = 0$ lies in the plane with unit normal vector $\boldsymbol{n}$ and signed distance c from the origin. The plane is fitted to the points $\boldsymbol{r}_i,\ i = 1, \dots, N$, by minimizing the following objective function:

$$S = \sum_{i=1}^{N} w_i {d_i}^2,$$

where d_i is the distance from the point $\boldsymbol{r}_i = (x_i, y_i, z_i)^{\mathsf{T}}$ to the plane and w_i is its weight. The weights are defined by:

$$w_i \propto 1/\sigma_i^2, \quad \sum_{i=1}^{N} w_i = 1.$$

The solution to this minimization problem is a plane with a normal vector $\boldsymbol{n}$ that is the unit eigenvector corresponding to the smallest eigenvalue of the weighted sample covariance matrix $\boldsymbol{A}$, defined as:

$$\boldsymbol{A} = \sum_{i=1}^{N} w_i(\boldsymbol{r}_i - \boldsymbol{r}_0)(\boldsymbol{r}_i - \boldsymbol{r}_0)^{\mathsf{T}},$$

where $\boldsymbol{r}_0$ is the weighted average or center of gravity:

$$\boldsymbol{r}_0 = \sum_{i=1}^{N} w_i \boldsymbol{r}_i.$$

Given $\boldsymbol{n}$, c is computed by:

$$c = -\boldsymbol{n}^{\mathsf{T}}\boldsymbol{r}_0.$$

The parameters $\boldsymbol{n}$ and c of the plane can then be mapped to a set of parameters of the corresponding circle in the (u, v)-plane [9].

We have followed Chernov's [7] suggestion of centering and scaling the measurements before mapping to the paraboloid, in order to achieve invariance of the fit under translations and similarities [6]. Centering is performed by subtracting the average:

$$u_{\mathrm{c},i} = u_i - \bar{u}, \quad v_{\mathrm{c},i} = v_i - \bar{v}, \quad i = 1, \dots, N$$

with

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i, \quad \bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i.$$

The centered measurements $u_{\mathrm{c},i}$ and $v_{\mathrm{c},i}$ are arranged in column vectors $\boldsymbol{u}_{\mathrm{c}}$ and $\boldsymbol{v}_{\mathrm{c}}$. Centered and scaled measurement vectors $\boldsymbol{u}_{\mathrm{cs}}$ and $\boldsymbol{v}_{\mathrm{cs}}$ are then obtained by:

$$s = b/\sqrt{(\boldsymbol{u}_{\mathrm{c}}^{\mathsf{T}}\boldsymbol{u}_{\mathrm{c}} + \boldsymbol{v}_{\mathrm{c}}^{\mathsf{T}}\boldsymbol{v}_{\mathrm{c}})/N}$$

$$\boldsymbol{u}_{\mathrm{cs}} = s \cdot \boldsymbol{u}_{\mathrm{c}}, \ \boldsymbol{v}_{\mathrm{cs}} = s \cdot \boldsymbol{v}_{\mathrm{c}}$$

where s is the scaling factor and b an arbitrary, preselected constant [6].

3. Simulation study in a generic cylindrical detector

We have simulated a generic type of a cylindrical detector system embedded in a perfectly homogeneous magnetic field, so that the track model in the bending plane of the particles is a circle. The simulated track sample is the same as the one used in [6]: 10 000 tracks coming from the origin with radii of curvature in a range from about 1.5 m to about 750 m. This corresponds to arcs between less than 0.1° and about 20°, following a reasonably flat distribution in this range. There are between 10 and 12 hits per track, and the single hit resolution varies between 0.1 and 1.5 mm. The measurement error in the radial direction is assumed to be negligible. We assume no background and thereby implicitly a perfect pattern recognition. The simulation does not include material and detector effects such as multiple scattering, energy loss and sensor misalignment. Measurements in different layers are therefore statistically independent.

We have compared the performance of the modified Riemann fit with a number of other circular track fitting algorithms by considering the mean-square error (MSE) of the residuals $\boldsymbol{\delta}$ of the track parameters, i.e. the estimated track parameters minus the true ones. The MSE is defined by:

$$\mathsf{MSE}[\boldsymbol{\delta}] = \det\left(\Sigma[\boldsymbol{\delta}] + \bar{\boldsymbol{\delta}}\bar{\boldsymbol{\delta}}^{\mathsf{T}}\right),$$

where $\Sigma[\boldsymbol{\delta}]$ is the sample covariance matrix and $\bar{\boldsymbol{\delta}}$ is the sample mean of the residuals. $\bar{\boldsymbol{\delta}}$ is the least-squares estimate of the bias of the track parameters.

Fig. 1 shows the MSE of the various estimators relative to the baseline, which is an iterative, non-linear least-squares approach using the Levenberg–Marquardt algorithm. Firstly, it can be seen that the modified Riemann fit performs better than the other non-iterative circle fitting algorithms, including the original Riemann track fit. The improvement in general grows with increasing measurement uncertainties. Secondly, the modified Riemann fit is seen to be virtually as precise as the Kalman filter, the global linear least-squares estimator and the non-linear method for the entire range of measurement uncertainties.

A similar plot of the generalized variance, defined as the determinant of the sample covariance matrix, shows no visible difference from Fig. 1. From this we conclude that the bias of all estimators is negligible compared to their spread. For a general error and bias analysis of a wide range of circle fitting algorithms, see [10].

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