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Charged particle tracking without magnetic field: Optimal measurement of track momentum by a Bayesian analysis of the multiple measurements of deflections due to multiple scattering

Mikael Frosini, Denis Bernard *

LLR, Ecole Polytechnique, CNRS/IN2P3, 91128 Palaiseau, France

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ABSTRACT

We revisit the precision of the measurement of track parameters (position, angle) with optimal methods in the presence of detector resolution, multiple scattering and zero magnetic field. We then obtain an optimal estimator of the track momentum by a Bayesian analysis of the filtering innovations of a series of Kalman filters applied to the track.

This work could pave the way to the development of autonomous high-performance gas time-projection chambers (TPC) or silicon wafer γ-ray space telescopes and be a powerful guide in the optimization of the design of the multi-kilo-ton liquid argon TPCs that are under development for neutrino studies.

1. Introduction

1.1. γ-ray astronomy

A huge effort is in progress to design γ-ray telescopes able to bridge the sensitivity gap that extends between the upper end of the high-sensitivity energy range of past and present X-ray telescopes and the lower end of the high-sensitivity energy range of the Fermi-LAT telescope, that is, approximately 0.1–100 MeV.

On the low-energy side of the gap, tracking of the electron issued from the first Compton scattering of an incident photon enables a major improvement of the precision of the reconstruction of the direction of the incident photon ([1] and references therein) that induces an impressive improvement of the true-photon-background rejection and therefore of the point-like-source sensitivity. A serious limitation of that ETCC (electron tracking Compton camera) scheme arises though, as the effective area undergoes a sharp drop for photon energies above 0.5 MeV, due to the fact that the recoil electron can exit on the side and escape energy measurement [1]: electron momentum measurement inside the time projection chamber (TPC) itself is highly desirable.

On the high-energy side of the gap, novel approaches improve the sensitivity by improving the single-photon angular resolution by using converters having a lower-Z than that of the tungsten plates of the EGRET/Fermi-LAT series. Using a series of silicon wafer active targets placed at a distance of each other, at the same time the material in which the photon converts and in which the tracks are tracked, enables an improvement of $\approx$ a factor of three in the angular resolution at 100 MeV with respect to the LAT [2–9] at the cost of a lower average active target density. Similar values of the angular resolution are achieved using a high-spatial-resolution, homogeneous, high-density material such as an emulsion [10].

If the trend to lower densities is pushed to the use of a gaseous detector, the angular resolution with respect to the LAT can increase up to a factor of ten at 100 MeV [11] and the single-track angular resolution is so good that the azimuthal angle of the e^+e^- pair can be measured with a good enough precision to enable the measurement of the linear polarization fraction of the incident radiation [12–14]. Gas detectors enable the detection of low-energy photons close to the pair-creation threshold where most of the statistics lie for cosmic sources (Fig. 1 of [15]), something which is critical for polarimetry.

Astrophysicists also need to measure the energy of incoming photons and therefore the momentum of the conversion electron(s). This can be achieved using a number of techniques.

- In a calorimeter, the total energy of the photon is absorbed and measured.
- In a magnetic spectrometer, the trajectory of a particle with electric charge q and momentum p in a magnetic field B is curved with a curvature radius $\rho = p/(qB)$: from a measurement of ρ, one

* Corresponding author.
E-mail address: denis.bernard@in2p3.fr (D. Bernard).

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obtains a measurement of p and in the end of the photon energy E.

- In a transition radiation detector (TRD), the energy of the radiation emitted in the forward direction by a charged particle at the interface between two media that have different refraction indices is proportional to the Lorentz factor γ of the particle, enabling a direct measurement. The low number of emitted photons per track per interface has lead to the development of multi-foil systems that suffer destructive interference at high energies. Appropriate configurations have showed saturation values larger than $\gamma \approx 10^4$, which implies that a measurement can be done up to a photon energy of ≈10 GeV [16].

The low-density active targets that have been considered above can provide a large effective area telescope only with a large volume ($\mathcal{O}(\mathrm{m}^3)$) and therefore the mass of the additional device used for energy measurement is a serious issue onboard a space mission. In this document we first address the performance of the track momentum measurement from measurements of the angular deflections of charged tracks due to multiple scattering during the propagation in the tracker itself.

1.2. Large noble-liquid TPCs for neutrino physics

Neutrino oscillation is a well established phenomenon and several experiments are being prepared with the goals:

- To test the occurrence of CP violation in the neutral lepton sector, i.e. to measure the only free complex phase δ of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix with enough precision to determine its non-compatibility with zero,
- To determine unambiguously the 3 neutrino mass ordering, i.e. to solve the sign ambiguity of the square mass difference Δm^2_{31}.

Not only the (vacuum propagation) phase term that involves δ changes sign upon $\nu \leftrightarrow \bar{\nu}$ exchange, but the term that describes the interaction with matter changes sign too as our Earth contains much more electrons than positrons. "In the few-GeV energy range, the asymmetry from the matter effect increases with baseline as the neutrinos pass through more matter, therefore an experiment with a longer baseline [is] more sensitive to the neutrino mass hierarchy. For baselines longer than ≈1200 km, the degeneracy between the asymmetries from matter and CP-violation effects can be resolved" [17]. Large distances imply low fluxes, that is, huge detectors and, to match the $\sin(\Delta m L/4E_\nu)$ oscillation function, high-energy neutrinos. So we should be prepared to measure the momentum of high-momentum muons in huge non-magnetized detectors such as liquid argon (lAr) TPCs.

The DUNE experiment expects to be able to measure muon momenta with a relative precision of ≈18% [18], based on a past ICARUS work [19]. They "anticipate that the resolution will deteriorate for higher-energy muons because they scatter less", though. Given the $\mathrm{d}E/\mathrm{d}x$ of 0.2 GeV/m of minimum ionizing particles in lAr, a typical 6 GeV/c muon produces a long track: it should be interesting to study to what extent an optimal analysis of the thousands of measurements per track, at their ≈3 mm sampling pitch, can do better.

1.3. Track momentum measurement from multiple scattering

The measurement of track momentum using multiple scattering was pioneered by Molière [20] and has been used since, in particular in the context of emulsion detectors (recent accounts can be found in [21,22]).

In a practical detector consisting of N detection layers, the precision of the deflection measurement and therefore of the momentum measurement is affected by the precision, σ, of the measurement of the position of the track when crossing each layer: the combined square deflection angle summed up over the whole track length therefore includes contributions from both the scattering angle and the detector precision. Bernard has optimized the longitudinal "cell" length over which each deflection angle is measured [11] and obtains a value of the relative momentum precision σ_p/p that scales as $p^{1/3}$, but the fact that the track position precision can improve when the cell length is extended and several measurements are combined was not taken into account in [11]. In the present document we study an optimal method of momentum measurement with a tracker that has a finite (non zero) precision.

In Section 2 we revisit optimal tracking methods in a context where the momentum of the particle is known. This allows us to present concepts and notations that are used later in the paper. We also extend the results published in the past by the use of more powerful methods.

The optimal precision of track measurements obtained in Section 2 can be obtained by performing the fit with a Kalman filter (KF), a tool that was imported in our field by Frühwirth [23]. In Section 3 we give a brief summary of Kalman filter tracking in a Bayesian formalism. In magnetic spectrometers, the particle momentum takes part both in the particle state vector through the curvature of the trajectory and in the magnitude of multiple scattering. The precision of the magnetic measurement is most often so good that the momentum can rightfully be considered as being perfectly known in the expression of the multiple scattering. In our case of a zero magnetic field, it is not the case. A Kalman filter is the optimal estimate for linear system models with additive white noise, such is the case for multiple scattering (process noise) and detector precision (measurement noise), but at the condition that the optimal Kalman gain be used in the expression, that is, that the track momentum be known. In Section 4, we use the Bayesian method developed by Matisko and Havlena [24] to obtain an optimal estimator of the process noise covariance, and therefore of the track momentum.[1] We implement this method and characterize its performance on Monte Carlo (MC) simulated tracks. We check that the momentum measurement is unbiased within uncertainties. We obtain a heuristic analytical expression of the relative momentum uncertainty.

Numerical examples are given for a homogeneous gas detector such as an argon TPC and for a silicon-wafer detector:

- TPC gas, argon, 5 bar, $\sigma = l = 0.1$ cm, $L = 30$ cm [12];
- Silicon detector $N = 56$, $\Delta x = 500$ μm-thick wafers spaced by $l = 1$ cm, with a single point precision of $\sigma = 70$ μm [8].

In this work a number of approximations are done: only the Gaussian core of the multiple-scattering angle distribution is considered and the non-Gaussian tails due to large-angle single scatters are neglected. The small logarithmic correction term in the expression of the RMS multiple scattering angle, θ_0, is neglected

$$\theta_0 \approx \frac{p_0}{\beta p}\sqrt{\frac{\Delta x}{X_0}}, \tag{1}$$

where $p_0 = 13.6$ MeV/c is the "multiple-scattering constant", Δx is the matter thickness through which the particle propagates and X_0 is its radiation length (Eqs. (33.14), (33.15), (33.17) of [26]). In the case of a homogeneous detector, the thickness of the scatterer is equal to the length of the longitudinal sampling, $l = \Delta x$. We assume relativistic particles ($\beta \approx 1$) without loss of generality. Only the first-order term (angle deflection) of multiple scattering is taken into account which is legitimate for the thin detectors considered here; the 2nd-order transverse displacement (Eq. (33.19) of [26]) is neglected. Continuous ($\mathrm{d}E/\mathrm{d}x$) and discrete (Bremsstrahlung radiation) energy losses are also neglected. In TPCs in which the signal is sampled, most often the electronics applies a shaping of the pulse before digitization, that creates a short scale longitudinal correlation between successive measurements that we neglect too. Also the limitations of pattern recognition, that is,

[1] Attempts of estimation of track momenta based on the use of a Kalman filter have been performed in the past, with little success. The un-validated un-characterized study of Ref. [25], for example, shows a poor relative resolution of σ_p/p = 30%–40% and that does not vary with the true particle momentum between 50 MeV/c and 2 GeV/c, which is a bad symptom.