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Dynamics of the off axis intense beam propagation in a spiral inflector

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ABSTRACT

In this paper the dynamics of space charge dominated beam in a spiral inflector is discussed by developing equations of motion for centroid and beam envelope for the off axis beam propagation. Evolution of the beam centroid and beam envelope is studied as a function of the beam current for various input beam parameters. The transmission of beam through the inflector is also estimated as a function of the beam current for an on axis and off axis beam by tracking a large number of particles. Simulation studies show that shift of the centroid from the axis at the inflector entrance affects the centroid location at the exit of the inflector and causes reduction in the beam transmission. The centroid shift at the entrance in the horizontal plane (h plane) is more critical as it affects the centroid shift in the vertical plane (u plane) by a large amount near the inflector exit where the available aperture is small. The beam transmission is found to reduce with increase in the centroid shift as well as with the beam current.

1. Introduction

The requirement of high power and high quality beams in many applications such as accelerator driven systems, spallation neutron sources, nuclear waste transmutation etc. has generated a considerable interest in the development of high current cw accelerators [1–3]. The development of a 10 MeV, 5 mA compact proton cyclotron at VECC [4] is a part of the considerably larger activity undergoing in the field of high intensity accelerator development for ADSS applications. The main aim is to identify and settle various physics and technological issues associated with the transport, injection and acceleration of high intensity proton beam in a cyclotron. In this connection we have developed 2.45 GHz microwave ion source which is presently operating and delivering 12 mA of proton beam at 80 keV with just 400 W of microwave power [5,6]. The extracted beam from the ion source has been transported up to 3 m in a solenoid based beam transport line and several experiments have been performed to characterize the beam. The beam from the ion source will be injected axially using a spiral inflector [7–11] in the central region of the cyclotron. Spiral inflector is a three electrode device consisting of a pair of biased electrodes housed into a grounded shield, being the third electrode. Its task is to inflect the axially injected beam in the median plane of the cyclotron and place the beam on proper orbit without any degradation in the quality of the beam. For a given injection energy and central magnetic field, the geometry of the spiral inflector depends mainly on two adjustable parameters, the inflector height and the tilt angle. For a fixed geometry, the beam properties at the exit of the inflector strongly depend on the condition of the beam at the entrance of the inflector [12–14].

It is well known that in high current accelerators and beam transport systems, the displacement of the beam with respect to the ideal propagation axis has significant effect on the dynamics of beam [15,16]. In such cases, the control of the beam centroid becomes difficult. The transport and placing the beam precisely on the desired location turn out to be a challenging task. Any mismatch of the beam at the entrance of the inflector causes degradation in its quality and transmission loss. It also causes mismatch with the acceptance at the central region of the cyclotron which leads to the envelope oscillations and amplitude growth of the beam during the acceleration. This effect becomes more severe when the beam current is high. Though, many authors [17–19] have discussed the beam transport through the spiral inflector including the effect of space charge, however, to the best of our knowledge, no attempt has so far been made to investigate the dynamics of the intense beam when the input beam is off-centered with respect to the central axis of the inflector. The above mentioned facts and the recent requirement of high intensity beam injection into the cyclotron have motivated us to investigate the behaviour of off axis intense beam propagation through the spiral inflector.

In this work, we have first obtained the equations of paraxial rays and centroid motion and then derived the beam envelope equations for a continuous intense off-axis beam through the spiral inflector using the kinetic model based on the nonlinear Vlasov–Maxwell equations [15,20]. Evolutions of beam centroid and beam envelope are studied as a function of beam current for various input beam parameters. The transmission of beam through the inflector has been studied as a

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function of the beam current for the on axis and off axis beams at the entrance.

2. Theoretical analysis

We consider a space charge dominated uniform density beam propagating through a spiral inflector with average axial velocity v_0. A schematic of the spiral inflector and two coordinate systems used in the development of equations of motion is shown in Fig. 1. In the present analysis it is assumed that trajectories of the beam particles remain very close to the axis, transverse beam sizes are small compared to the spacing between the electrodes and the transverse velocities are very small as compared to the average axial velocity of the particles.

2.1. Central ion trajectory

The parametric equations for the central ion trajectory in a spiral inflector have been discussed in detail in the literature [7,8]. We will rewrite them here directly. These equations can be easily obtained by solving the well known Lorentz force equation in the applied electric and magnetic fields. These are:

$$x_c(s) = \frac{A}{2}\left[\frac{2}{1-4K^2} + \frac{\cos[(2K-1)b]}{2K-1} - \frac{\cos[(2K+1)b]}{2K+1}\right] \quad , \qquad 1(a)$$

$$y_c(s) = -\frac{A}{2}\left[\frac{\sin[(2K+1)b]}{2K+1} - \frac{\sin[(2K-1)b]}{2K-1}\right] \quad , \qquad 1(b)$$

$$z_c(s) = A(1-\sin b),\ 0 \le b \le \pi/2. \qquad 1(c)$$

Here we have used a fixed right handed Cartesian coordinate system x, y and z with unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$ respectively. Its origin lies on the cyclotron axis in the median plane (x-y plane) and the z axis is vertically opposite to the direction of the incoming ion. The uniform magnetic field B_0 is opposite to the z direction. The electric field E_0 is along the x direction at the entrance of the inflector and remains always perpendicular to the central trajectory. In the above equations $b = (v_0 t/A) = (s/A)$ represents the instantaneous angle of the velocity vector with the vertical and s measures the distance along the reference trajectory with $s = 0$ at the inflector entrance and $S = \pi A/2$ at the exit. The shape parameter K, given by,

$$K = \frac{A}{2R_m} + \frac{k'}{2} \quad , \qquad (2)$$

includes the magnetic radius $R_m = p/qB_0$ and electric radius $A = mv_0^2/qE_0$ and a free tilt parameter k' where p is the momentum and q is the charge of the ion. Parameters A and k' play a key role in the design of the spiral inflector and help to place the beam in the central region for proper orbit centering. They also influence the beam properties at the exit of the spiral inflector [14].

2.2. Paraxial equations of motion with space charge

In order to develop the paraxial equations of motion of an off axis beam propagation in a spiral inflector we use the optical coordinate system u, h and v with basis vectors $\hat{u}$, $\hat{h}$ and $\hat{v}$ which moves in space along with the central ion trajectory. Here u, h and v denote the coordinates of a paraxial ray. Vector $\vec{v}$ is along the direction of the velocity of the beam, vector $\vec{h}$ is parallel to the median plane (x-y plane) and vector $\vec{u} = \vec{h} \times \vec{v}$ (Fig. 1). Under the paraxial approximation the equations of motion of a particle of charge q and mass m in the combined self-fields and applied electric and magnetic fields in the spiral inflector can be obtained from the Hamiltonian H (normalized to mv_0^2) [21]

$$H = \tfrac{1}{2}\left[\left(p_u - \tfrac{FC}{A}h\right)^2 + \left(p_h + \tfrac{FC}{A}u\right)^2 + \left(p_v + \tfrac{2}{A}u - \tfrac{2FS}{A}h\right)^2\right] - \tfrac{\xi}{2A^2}(u - k'Sh)^2 - \tfrac{u^2}{2A^2} - \tfrac{Kk'}{A^2}(C^2u^2 + h^2) + \tfrac{FS}{A^2}uh + \phi^{sc} \quad , \qquad (3)$$

as

$$u' = p_u - \frac{FC}{A}h \quad , \qquad (4a)$$

$$p_u' = -\frac{3 - \xi + (F^2 - 2K\,k')C^2}{A^2}u + \frac{3FS - k'\xi\,S}{A^2}h - \frac{FC}{A}p_h - \frac{2}{A}p_v + \frac{\partial\phi^S}{\partial u} \quad , \qquad (4b)$$

$$h' = p_h + \frac{FC}{A}u \quad , \qquad (4c)$$

$$p_h' = \frac{3FS - k'\xi\,S}{A^2}u + \frac{FC}{A}p_u + \frac{2FS}{A}p_v - \frac{(1+3S^2)F^2 - 2Kk' - k'^2\xi\,S^2}{A^2}h + \frac{\partial\phi^S}{\partial h} \quad , \qquad (4d)$$

$$v' = \frac{2}{A}u - \frac{2FS}{A}h + p_v \quad , \qquad (4e)$$

$$p_v' = 0 \quad , \qquad (4f)$$

where
$C = \cos(s/A)$, $S = \sin(s/A)$,

$$\xi = \frac{1 + 2K\,k'\,S^2}{1 + k'^2S^2},\ F = K + \frac{k'}{2}. \qquad (5)$$

Here p_u, p_h and p_v are the canonical momentum normalized to mv_0 and $\phi^s(u, h, s)$ is the space charge potential normalized to mv_0^2/q.

2.3. Motion of beam centroid

In the paraxial approximation, the dynamics of the beam particles in transverse phase space (u, p_u, h, p_h) can be described self-consistently using the nonlinear Vlasov-Maxwell equations for a distribution function $f(u, p_u, h, p_h, s)$ and the normalized self-field potential $\phi^s(u, h, s)$ as [20],

$$\frac{\partial f}{\partial s} + u'\frac{\partial f}{\partial u} + h'\frac{\partial f}{\partial h} + p_u'\frac{\partial f}{\partial p_u} + p_h'\frac{\partial f}{\partial p_h} = 0 \quad , \qquad (6a)$$

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial h^2}\right)\phi^s(u, h, s) = -\frac{q^2\,n(u, h, s)}{\varepsilon_0 mv_0^2} \quad , \qquad (6b)$$

where $n(u, h, s) = \iint f\, dp_u\, dp_h$ is the number density of beam particles. Let $\chi(u, p_u, h, p_h, s)$ be any function of phase space variable, then the statistical average of χ is defined by

$$\langle\chi\rangle = \frac{1}{n_0}\iint f\,\chi\, du\, dh\, dp_u\, dp_h, \qquad (7)$$

where $n_0 = \iint f\ du\, dh\, dp_u\, dp_h = \text{const.}$, is the number of beam particles per unit axial length.

The beam centroid is defined by the first moment of f, i.e. $u_c = \langle u\rangle$, $h_c = \langle h\rangle$ and $v_c = \langle v\rangle$. Using Eqs. (4) and averaging over the distribution function we obtain

$$\langle u\rangle' = \left\langle u'\right\rangle = \langle p_u\rangle - \frac{FC}{A}\langle h\rangle, \qquad (8a)$$

$$\left\langle p_u\right\rangle' = \left\langle p_u'\right\rangle = -\tfrac{3 - \xi + (F^2 - 2Kk')C^2}{A^2}\left\langle u\right\rangle + \tfrac{3FS - k'\xi S}{A^2}\left\langle h\right\rangle - \tfrac{FC}{A}\left\langle p_h\right\rangle - \tfrac{2}{A}\left\langle p_v\right\rangle + \left\langle \tfrac{\partial\phi^S}{\partial u}\right\rangle \quad , \qquad (8b)$$

$$\left\langle h\right\rangle' = \left\langle h'\right\rangle = \left\langle p_h\right\rangle + \frac{FC}{A}\left\langle u\right\rangle \quad , \qquad (8c)$$