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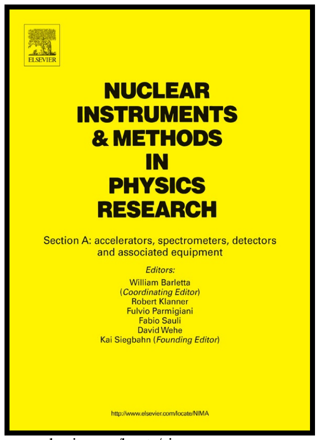

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Optical beam with vortices: a first order paraxial analysis

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We analyze different forms of light pulses with Orbital Angular Momentum based on two-index Hermite polynomials, their Poynting vector and angular momentum in paraxial propagation. We extend the analysis at first order in the paraxial approximation, calculating the transverse component of the Poynting vector and the longitudinal component of the angular momentum, which give the rotation of the energy flux around the vortices.

I. INTRODUCTION

Helically phased light beams and optical vortices, in the visible and infrared wavelength range, carry orbital angular momentum (OAM) [1] directly transferable to atoms, molecules and nanostructures, allowing applications as optical data transmission, crystal micromanipulation, microscopy, detection of spinning terrestrial and astronomical objects [2-7] and acceleration of particles [8]. At shorter wavelengths OAM beams could be employed in photoionization experiments, where the violation of the dipolar selection rules could excite new effects [9], in x-ray magnetic circular dichroism, where quadrupolar and dipolar transitions could be unfolded [10], and in the resonant inelastic x-ray scattering of many materials, where the molecules with vibrational freedom degrees could resonate in the vortex, providing informations on their structure [11].

Methods for generating OAM beams in the visible range consist in sending the light through a fork hologram or a spiral phase mask, techniques that has also been tried in the XUV or x-ray range [12-13], with limitations associated to the damage threshold of the optical elements of the beam line at large intensity and to the fabrication of high quality optical surfaces. Therefore, the generation of OAM pulses at short wavelengths and high brilliance is still an open problem.

It has been predicted [14] and then demonstrated experimentally [15] that OAM can be obtained in the regime of spontaneous emission radiation from undulators with variable polarization. Other approaches aimed at generating vortex beams at short wavelengths with Free-Electron Lasers have been proposed [16-19]. Moreover, the frequency upshift of OAM optical beams occurring in the Compton back-scattering [20-21] has been proposed for reaching the hard x-ray range. In most of these techniques, the starting point is the availability of OAM laser pulses in the infrared, optical or ultraviolet frequency ranges.

In this paper, we will consider this subject, organizing the work in the following way: in the second section, the properties of the multi-index Hermite polynomials and their application to the case of optical modes with vortices is discussed in the paraxial approximation. In the following, we discuss the necessity to overcome the customary paraxial limit and we show the fields at first order in the paraxial approximation. The behavior of Poynting vector and orbital momentum at first order are analyzed. Finally, we end with comments and conclusions.

II. MATHEMATICAL ANALYSIS OF OAM LIGHT BEAMS

Radiation with angular momentum with a pair of vortices can be studied at zero order in the paraxial approximation by assuming the scalar electric field:

$$E(x,y,z,t) = f(z-ct)E_{m,n}(x,y,z)e^{i(\omega t - k_z z)} \tag{1}$$

where ω and k_z are angular frequency and wave number, $f(z-ct)$ is an arbitrary function and:

$$E_{m,n}(x,y,z) = \pi\left(\frac{w_0}{w_z}\right)^2 H_{m,n}(X,\Lambda,Y,\Gamma|T)e^{-\frac{x^2+y^2}{2w_z^2}} \tag{2}$$

with $X = \frac{1}{w_0}\left[(\frac{w_0}{w_z})^2(x+i\varepsilon_1 y)-(x_1+i\varepsilon_1 y_1)\right]$, $Y = \frac{1}{w_0}\left[(\frac{w_0}{w_z})^2(x+i\varepsilon_2 y)-(x_2+i\varepsilon_2 y_2)\right]$, $\Lambda = \frac{i}{2}(1-\varepsilon_1^2)\frac{\lambda z}{w_z^2}$, $\Gamma = \frac{i}{2}(1-\varepsilon_2^2)\frac{\lambda z}{w_z^2}$ and $T = i\frac{\lambda z}{w_z^2}(1-\varepsilon_1\varepsilon_2)$. Moreover (x_1, y_1) and (x_2, y_2) are the centers of the two vortices, $w_z^2 = w_0^2(1+i\frac{z}{z_R})$, w_0 and $z_R = kw_0^2/2$ are the transverse and longitudinal characteristic lengths of the Gaussian host beam and $\varepsilon_{1,2}$ are the topological charges [22].

In the definition of the two-index Hermite polynomials [23-25]:

$$H_{m,n} = m!n!\sum_{r=0}^{min(n,m)}\frac{\tau^r H_{m-r}(\xi,a)H_{n-r}(\eta,b)}{r!(m-r)!(n-r)!}, \tag{3}$$

$H_k(\varsigma,\alpha) = k!\sum_{r=0}^{[k/2]}\frac{\alpha^r\varsigma^{k-2r}}{r!(k-2r)!}$ are the Hermite-Kampé de Fériet (H-KdF) polynomials.

When $\varepsilon_1^2 = \varepsilon_2^2 = 1$, and $\tau = 0$, eq. (3) leads to [22]:

$$E_{m,n}(x,y,z=0) = \pi e^{-\frac{x^2+y^2}{2w_0^2}}\frac{X_0^m Y_0^n}{w_0^{m+n}} \tag{4}$$