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Discussion and analytical test for inclusion of advanced field and boundary condition in theory of free electron lasers

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ABSTRACT

By the covariant statement of the distance in space-time separating transmitter and receivers, the emission and absorption of the retarded and advanced waves are all simultaneous. In other words, for signals carried on electromagnetic waves (advanced or retarded) the invariant interval $(cdt)^2 - dr^2$ between the emission of a wave and it's absorption at the non-reflecting boundary is always identically zero. Utilizing this principle, we have previously explained the advantages of including the coherent radiation reaction force as a part of the solution to the boundary value problem for FELs that radiate into “free space” (Self Amplified Spontaneous Emission (SASE) FELs) and discussed how the advanced field of the absorber can interact with the radiating particles at the time of emission. Here we present an analytical test which verifies that a multilayer mirror can act as a band pass filter and can contribute to microbunching in the electron beam. Here we will discuss motivation, conditions and requirements, and method for testing this effect.

1. Introduction and background

Interaction of a charged particle with its own radiation, radiation reaction, has been a subject of many controversial discussions and is recognized as one of the oldest open questions in classical electrodynamics (CED). Many reviews of efforts made to address the issue of the conservation of energy and radiation reaction force have been published (i.e. [1]). It is often argued that the problem of conservation of energy in CED lies with the failure of classical physics at small scales. A thorough study of the problem of coherent radiation in classical electrodynamics with this respect, however, refuted the above argument. We were able to show that Maxwell energy integral test for coherent emission fails at all wavelengths and physical scales including radio and microwave wavelengths [2]. A summary of this test is demonstrated for the cases shown Fig. 1.a and b. in Figs. 2 and 3.

Consider two coherently oscillating charged particles in the nonrelativistic regime. When in equilibrium, the electromagnetic time-averaged energy stored within a spherical shell surrounding periodically oscillating charges is constant. Therefore the time-averaged surface integral of the Poynting vector must be equal to the time-averaged volume integral of $\mathbf{E}{\cdot}\mathbf{j}$ within the sphere of integration.

As shown in Figs. 2 and 3 Sommerfeld's “retarded only” model (including only the retarded Lienard-Wiechert potential in the calculation of $\mathbf{E}{\cdot}\mathbf{j}$) does not pass the Maxwell integral test of energy conservation. Dirac's model however, comes close in the limiting case of distances larger than $\sim 4\lambda$ and matches the Maxwell prediction perfectly when r=0. On the other hand, a time-symmetric electrodynamics (TSE & M) model, developed based on the work done by Wheeler and Feynman [3], is a perfect match for all values of r. For detailed discussion and analysis of the summary presented here see Ch. 8 of [4].

This result suggests that, for sources with a large number of coherently oscillating particles in free space (SASE FELs), the advanced field of the absorber could play an important role in enhancing the output. Additionally, studying the role of the advanced field can provide us with a better understanding of the fundamentals of the process of coherent radiation, particularly with regard to the role of radiation reaction.

2. More on time-symmetric electrodynamics

There is no doubt that the result discussed above is conceptually demanding as our classical experience is limited to the retarded field. In the past few decades, however, there have been analyses and experiments that indirectly support the time-symmetry promised by Maxwell equations. A few of these experiments/analysis are based on the interaction of light and matter. Here we give a short background and discuss these experiments/analyses briefly. These experiments/analyses have played an important role in our approach to the question of conservation of energy for coherently oscillating charged particles

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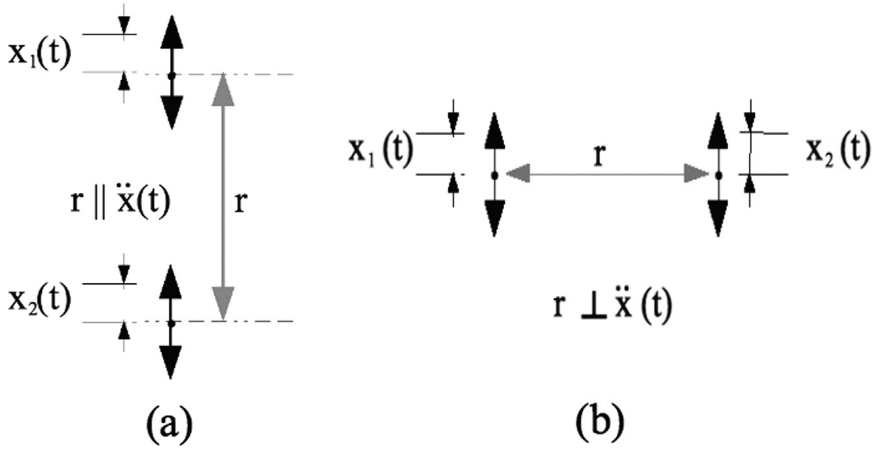

Fig. 1. Two coherently oscillating charged particles with distance *r* between the centers of their oscillation, and amplitude of oscillation of *x*(*t*), where **r** is (a) parallel (b) normal to the direction of oscillation.

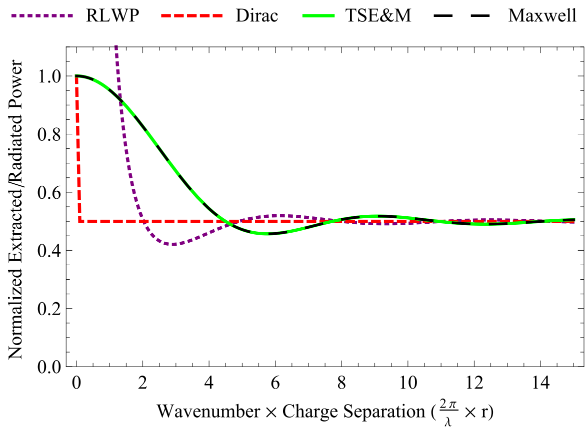

Fig. 2. Comparison of the normalized total radiated power from Maxwell's integral equations (Maxwell in black), the normalized amplitudes of those components of the fields acting on the individual charges in phase with their velocities calculated using the retarded only Lienard Wiechert potentials (RLWP in purple), the volume integral of **E**·**j** attributable to the Dirac's (in red), Wheeler and Feynman (TSE & M in green) coherent radiation force for two coherently oscillating charged particles (Fig. 1.a) showing dependence on the separation between the centers of oscillation of two charges. (The divergent field amplitudes at small charge separations compared to the wavelength is due to the charge's retarded Coulomb fields.).

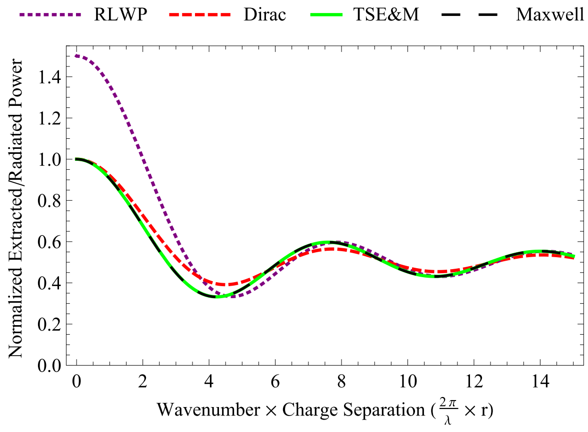

Fig. 3. Comparison of the normalized total radiated power from Maxwell's integral equations (Maxwell in black), the normalized amplitudes of those components of the fields acting on the individual charges in phase with their velocities calculated using the retarded only Lienard Wiechert potentials (RLWP in purple), the volume integral of **E**·**j** attributable to the Dirac's (in red), Wheeler and Feynman (TSE & M in green) coherent radiation force for two coherently oscillating charged particles (Fig. 1.b) showing dependence on the separation between the centers of oscillation of two charges.

(as examined in [2]) and have led us to propose the experimental setup with the high gain FEL which is explored in the following section.

2.1. *Photon Decay in Cavity QED experiments*

An illuminating example for us is a Cavity QED experiment, in which interaction of a single atom and a single photon studied. This interaction can be described in terms of a two-level approximation of Rabi oscillation. If the photon lifetime in the cavity is long enough it would be possible to measure its interaction with multiple single atoms that pass through the cavity. The photon would cause the passing atoms to be suspended in any arbitrary superposition of their ground or excited states leading to complex entangled states. One of the main breakthroughs was the observation of photon exchange between cavity mode tuned to resonance and single passing atoms in 1985 [5]. In this setup, a closed cylinder structure housed the cavity mirrors with small apertures for entry and exit of the atoms. The cavity significantly reduced the nonresonant decay of the photons. The closed structure of the cavity, however, was the cause for further technical difficulties in precision measurement. Atoms passing through tiny holes, very close to metal surfaces, produced additional fields that would disturb the atoms state. This problem was solved by using an open structure cavity with highly reflective mirrors. These mirrors were precision machined out of copper with a small amount of superconducting niobium doping on top of them. In this setup, photon lifetimes was extended to 130 ms allowing the photon to travel back and forth between the mirrors for 40,000 km. The feature that the open cavity, a reincarnation of Einstein-Bohr photon box, can reduce the photon decay similar to a closed cavity indirectly supports the time-symmetric property dis-cussed earlier.

2.2. *Laser normal mode expansion*

A highly debated analysis of laser and cavity equations of motion is the normal mode expansion by John Slater. This analysis is independent of the cavity shape and form. In his initial approximation, Slater assumes a lossless cavity with orthogonal modes. For conventional laser cavities, transverse and axial modes satisfy the general biorthogonal relation (not orthogonal). In other words, their forward and reverse operators are not Hermitian, however, the reverse propagator is the transpose of the forward propagator in the conventional laser cavity. Despite this, Slater's normal assumptions lead to ideal Hermite-Gaussian functions which are a very good approximation of the solutions for the system [6]. This feature also indirectly supports the time-symmetric properties discussed.