Nuclear Instruments and Methods in Physics Research A 865 (2017) 91–94

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Resonant excitation of betatron oscillations

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A R T I C L E I N F O

Keywords:
Plasma
Wakefield
Radiation
Undulator
Ion-channel laser
Accelerator

A B S T R A C T

Resonant excitation of betatron oscillations is a proposed mechanism for creating large amplitude, sinusoidal electron oscillations by overlaying a standard magnetic undulator and the radial electric fields of an ion channel to effectively create a very high strength undulator. By satisfying a resonance condition between the undulator and betatron periods, large transverse wiggling amplitudes are possible at short periods of oscillation. This skirts the existing limitations of magnetic undulators whose effective strength is constrained by the limits on high peak field values in a short (~cm) period. Additionally, such motion can be achieved with on axis beam injection and greater operational amplitude and polarization tunability than offered in other incarnations of the plasma wiggler. This technique provides a path to applications that demand high K, tunable, bright x-ray or gamma ray radiation such as high flux polarized positron production and a practical realization of the ion channel laser.

1. Introduction

Magnetic undulators and wigglers are devices traditionally based on a periodic lattice of transverse magnetic fields, which are employed to produce intense radiation by causing the oscillation of a relativistic electron bunch. Such devices are characterized by the strength parameter K:

$$K = \frac{eB\lambda_u}{2\pi m_e c} \approx 0.93\lambda_u[\text{cm}]B[\text{T}], \tag{1}$$

where e is the electron charge, B is the peak magnetic field of the lattice, λ_u is the period of the undulator, m_e is the electron mass, and c is the speed of light. Undulators are devices in the $K < 1$ regime which have a radiation spectrum with peaks at harmonics of the fundamental

$$\lambda_{rad} = \frac{\lambda_u}{2\gamma^2}, \tag{2}$$

where γ is the Lorentz factor. If $K \gg 1$, the device is termed a wiggler and its spectrum will approach that of a bending magnet with field $B = B_0/\sqrt{2}$ [1]. Since there are not spikes at the harmonic frequencies the spectra is characterized by the "cutoff energy", the maximum photon energy before the power (at fixed bandwidth) falls off exponentially:

$$\epsilon_c[\text{keV}] = 0.665B[\text{T}]E^2[\text{GeV}^2], \tag{3}$$

from which we see that the energy of the photons produced by a wiggler will have no explicit dependence on the period of the wiggler, λ_u.

The total radiated power (per electron, per period) for either case is given by

$$P_{rad} = \frac{e^4\gamma^2 B_0^2}{12\pi\epsilon_0 c m_e^2} = \frac{e^2 c\gamma^2 K^2 k_u^2}{12\pi\epsilon_0}, \tag{4}$$

where ϵ_0 is the permittivity of free space and k_u is the undulator wave number, $2\pi/\lambda_u$. In the last expression, we remove the explicit dependence on the B-field, which is advantageous for discussing alternative approaches to inducing wiggler motion, as we will do below. Thus we see that if we want to obtain high energy photons and high total radiated power we should seek to maximize the beam energy, K value, and k_u (i.e. minimizing λ_u). These devices ultimately are limited by restriction on magnetic fields (near 1 T) in standard magnetostatic devices with further practical limitations encountered in scaling the device to small undulator wavelength [2]. Superconducting magnets have offered improvements but such undulators having centimeter periods still tend to have fields on the order of a Tesla due to magnetic force handling issues. Permanent magnets, on the other hand, have expressly limited peak fields.

A compelling, recently introduced alternate technique that provides the functionality of a very high field undulator is the plasma wiggler [3–5]. Instead of using periodic magnetic fields however, the plasma wiggler relies on the nominally cylindrically symmetric, radial electric fields in the blowout region of an ion channel. To create an ion channel from a previously neutral plasma the plasma electrons near the beam

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http://dx.doi.org/10.1016/j.nima.2017.02.002
Received 6 October 2016; Received in revised form 1 February 2017; Accepted 1 February 2017
Available online 03 February 2017

axis are expelled by either a "driver" charged particle bunch or a high energy laser pulse. Here we concentrate on the case of a driving electron beam, termed plasma wakefield accelerator (PWFA). A witness bunch (in this case radiation producing) trails in the wake of the driver, placed near the zero crossing of the longitudinal electric field in the wake but transversely offset from the driver axis, so it experiences the radial electric fields in the blowout (electron-free) bubble region. This causes the witness bunch to oscillate at the betatron frequency with an amplitude equal to its incoming offset [3].

$$\omega_\beta = \frac{\omega_p}{\sqrt{2\gamma}}, \tag{5}$$

where the plasma frequency is defined

$$\omega_p = \sqrt{\frac{ne^2}{\epsilon_0 m_e}}, \tag{6}$$

where n is the electron number density. For typical laboratory plasmas with densities $10^{17} - 10^{19}\,\mathrm{cm}^{-3}$ betatron wavelengths can be on the order of 1 cm. The appeal of this technique is that the fields in the ion channel can be enormous, many GV/m, equivalent in force to tens or even hundreds of Tesla magnetic fields. Such values, for cm-scale period, place this type of undulator squarely in the $K \gg 1$, wiggler, regime. In this limit, the radiation will be broadband with critical energy [3]

$$\epsilon_c[keV] = 5\cdot 10^{-21}\gamma^2 n[\mathrm{cm}^{-3}]x_0[\mu\mathrm{m}] \tag{7}$$

where x_0 is the amplitude of the oscillation.

2. First order resonant betatron excitation dynamics

While the plasma wiggler is an attractive option for generating a high K device, it is difficult to integrate with existing techniques. For example, in the promising Trojan Horse injection scheme [6,7], small emittances are obtained by on-axis injection, and thus there is no resulting coherent betatron oscillation. Further, in the case of external injection, in which a large betatron amplitude can be introduced as an initial condition, it is extremely challenging to produce a helical orbit, which would be needed for, e.g., polarized positron production using circularly polarized gamma rays. To address the limitations both conventional and plasma-based undulators, consider combining the two. This involves producing a plasma inside an undulator, exciting a blowout regime PWFA with an intense drive beam, then resonantly exciting the plasma wiggler. Examining the first order dynamics of such a configuration (in the lab frame), we assume a simple superposition of the ion focusing electric fields and the magnetostatic field of the undulator. Explicitly, for the electric field, we assume a complete blowout of electrons from the relevant region (also assuming the witness trajectory does not exceed the bubble radius), that is

$$\mathbf{E} = -\frac{ner}{2\epsilon_0}\hat{r}. \tag{8}$$

The on-axis magnetic field of an undulator is

$$\mathbf{B} = B_0 \sin\left(\frac{2\pi z}{\lambda_u}\right)\hat{x}. \tag{9}$$

Based on the relativistic energy of the witness beam, we may also assume that γ is nearly constant and that motion outside the y-z plane can be neglected, permitting us to write:

$$\begin{cases} \ddot{x} = 0 \\ \ddot{y} = \dfrac{e}{\gamma m}\left(\dfrac{-ney}{2\epsilon_0} + B_0 \sin\left(\dfrac{2\pi z}{\lambda_u}\right)v_z\right). \\ \ddot{z} = \dfrac{e}{\gamma m}\left(-B_0 \sin\left(\dfrac{2\pi z}{\lambda_u}\right)v_y\right) \end{cases} \tag{10}$$

We assume the modification to v_z from the transverse oscillation can be ignored and that $z \approx ct$ giving a single, uncoupled differential equation. Simplifying with $\ddot{y} = c^2 y''$, $k_p = \sqrt{\frac{ne^2}{\epsilon_0 m_e c^2}}$, $k_\beta = \frac{k_p}{\sqrt{2\gamma}}$, and $k_u = \frac{2\pi}{\lambda_u}$:

$$y'' = -k_\beta^2 y + \frac{qB_0 c}{\gamma mc}\sin(k_u z). \tag{11}$$

Which takes the form of a driven harmonic oscillator, for which the resonant condition is $k_\beta = k_u$. For coaxial, on-axis witness beam injections, the first order trajectory is:

$$y(z) = \frac{qB_0}{\gamma mc}\frac{\sin(k_u z) - \frac{k_u}{k_\beta}\sin(k_\beta z)}{k_\beta^2 - k_u^2}. \tag{12}$$

When on resonance

$$y(z) = \frac{qB_0}{2\gamma mck_u^2}(\sin(k_u z) - k_u z\cos(k_u z)), \tag{13}$$

so the transverse amplitude is

$$\frac{qB_0}{2\gamma mck_u^2}\sqrt{1 + k_u^2 z^2}. \tag{14}$$

Defining $B_{\mathrm{eff}} = (B_0/2)\sqrt{1 + k_u^2 z^2}$, the transverse amplitude is written

$$\frac{qB_{\mathrm{eff}}}{\gamma mck_u^2}. \tag{15}$$

Thus the maximum excursion can be increased arbitrarily (within the confines of this simplified model) by tightly matching the plasma parameters to satisfy the resonant condition. Compare this to the expression for the peak amplitude in a pure magnetic undulator

$$\frac{qB_0}{\gamma mck_u^2}. \tag{16}$$

Saturation of the amplitude of resonantly driven betatron oscillations is physically obtained either by the encountering of non-ideal fields (i.e. when the amplitude of the electron response reaches the dimension of the electron-rarified bubble), or by a frequency mismatch. In the first case, one expects the resonant condition to be lost immediately upon loss of the linear focusing in the bubble, and the amplitude would be saturated. This is also a region of very nonlinear fields, and as such leaving the beam in a saturation scenario would significantly degrade its phase space quality. Saturation of the amplitude is also obtained when the linear resonance condition $k_\beta = k_u$ is not strictly satisfied. This scenario is discussed below.

2.1. Truncated RBE

There is no reason that this resonant condition must be maintained indefinitely; once the desired amplitude is reached the external magnetic field can be removed, leaving the particles to interact with a pure plasma wiggler having desired initial conditions. Such a configuration offers stability and tunable radiation. The "stability" is the tolerance of the system to being slightly off resonance, for example, due to shot-to-shot variations in plasma density. As shown above, if the resonance condition is not met the amplitude envelope is sinusoidal, but if the conditions are close, the envelope is approximately equal to the linear growth for small z. By truncating the external magnetic field early, relative to the expected error, the long term behavior of the beam envelope can be highly tolerant of mismatched parameters. The long term effect of a mismatch is shown in Fig. 1.

Compare this to Fig. 2 where the magnetic field is truncated after a few growth periods. In this case, the long term behavior is nearly indistinguishable for all cases with up to a 2% mismatch.

This truncation can also be used to deliver tunable radiation, even in the face of fixed beam energy and undulator length. Equation (14) shows that, even with everything else held constant, simply by varying