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A new three-dimensional track fit with multiple scattering

Niklaus Berger[a,b], Alexandr Kozlinskiy[a,b], Moritz Kiehn[a], André Schöning[a,*]

[a] Physikalisches Institut, Heidelberg University, Heidelberg, Germany
[b] Institut für Kernphysik and PRISMA cluster of excellence, Mainz University, Mainz, Germany

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ABSTRACT

Modern semiconductor detectors allow for charged particle tracking with ever increasing position resolution. Due to the reduction of the spatial hit uncertainties, multiple Coulomb scattering in the detector layers becomes the dominant source for tracking uncertainties. In this case long distance effects can be ignored for the momentum measurement, and the track fit can consequently be formulated as a sum of independent fits to hit triplets. In this paper we present an analytical solution for a three-dimensional triplet(s) fit in a homogeneous magnetic field based on a multiple scattering model. Track fitting of hit triplets is performed using a linearization ansatz. The momentum resolution is discussed for a typical spectrometer setup. Furthermore the track fit is compared with other track fits for two different pixel detector geometries, namely the Mu3e experiment at PSI and a typical high-energy collider experiment. For a large momentum range the triplets fit provides a significantly better performance than a single helix fit. The triplets fit is fast and can easily be parallelized, which makes it ideal for the implementation on parallel computing architectures.

1. Motivation

The trajectory of a free charged particle in a homogeneous magnetic field is described by a helix. The non-linear nature of the helix makes the reconstruction of the three-dimensional trajectory from tracking detector hits one of the main computational challenges in particle physics. To simplify the problem, the reconstruction is often factorized into a two-dimensional circle fit in the plane transverse to the magnetic field and a two-dimensional straight line fit in the longitudinal plane.[1] A non-iterative solution to this problem was given by Karimäki [1]. This simplified treatment however does not make full use of the available detector information and ignores correlations between the two planes, which can be large especially for small helix radii (low momentum particles) at small (large) polar angles $\vartheta \approx 0(\pi)$.

A further complication of the track reconstruction problem is the treatment of multiple Coulomb scattering (MS) in the detector material, which introduces correlations between the measurement points. This problem is addressed by Kálmán filters [2,3,4] and broken line fits [5–7] which both give a correct description of the track parameter error matrix. The methods however require computationally expensive matrix inversions and potentially multiple passes.

In modern semiconductor pixel trackers, extremely precise three-dimensional position information is available and tracking uncertainties are dominated by MS except at the very highest momenta. Usually most of the material causing the scattering is located in the sensors or very close to them (services, cooling, mechanics, etc.); therefore, the scattering planes usually coincide with the detection planes. This is our motivation for developing a new three-dimensional helix fit which treats MS in the detector as the only uncertainty. The resulting algorithm is based on triplets of hits which can be fit in parallel. The final result is then obtained by combining all triplets. The algorithm is computationally efficient and well suited for track finding. The first application of the algorithm is the all-pixel silicon tracker [8] of the Mu3e experiment [9].

2. Triplet track fit

The basic unit of the track fit is a triplet of hits in successive detector layers. In the absence of MS and energy losses, the description of a helix through three points requires eight parameters, namely a starting point (three parameters), an initial direction (two parameters), the curvature (one parameter) and the distances to the second and third points (two parameters). MS in the central plane requires two additional parameters to describe the change in track direction.[2] Three

* Corresponding author.

[1] In the right-handed coordinate system we define the B-field orientation along the z-axis; the azimuthal angle φ is defined in the transverse x–y plane and the polar angle ϑ is defined in the longitudinal z–s plane where s is the track length parameter.

[2] Two more parameters, describing a possible position offset at the central plane due to MS inside the material, can be ignored for typical silicon trackers, where the sensor thicknesses are much smaller than the distances between the detector layers.

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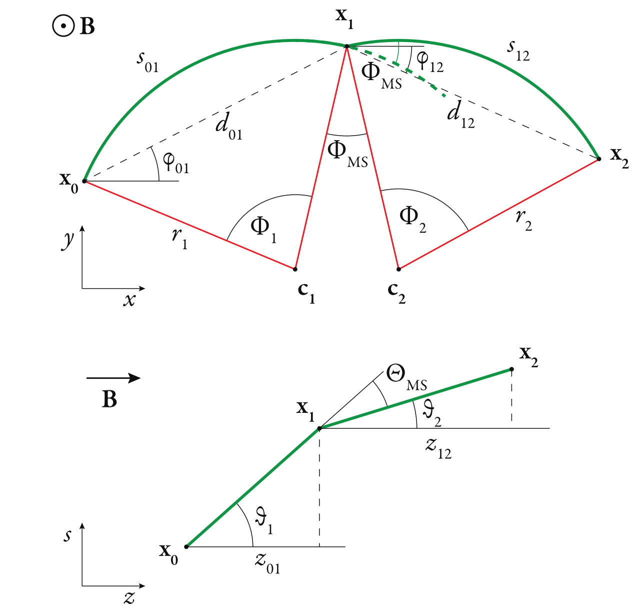

Fig. 1. Particle trajectory in a homogeneous magnetic field defined by a triplet of hits x_0, x_1 and x_2, with particle scattering at $\mathbf{x}_1$. The top view shows the projection to the plane transverse to the magnetic field, whereas the bottom view is a projection to the field axis-arclength (s) plane. r_1 and r_2 are the transverse track radii before and after the scattering process, s_{01} and s_{12} the corresponding arclengths and Φ_1 and Φ_2 the bending angles. d_{01} and d_{12} denote the transverse distances between x_0 and x_1, and x_1 and x_2, respectively. The azimuthal angles of the corresponding distance vectors are labeled φ_{01} and $\varphi_{12,}$ and Φ_{MS} is the transverse scattering angle. In the longitudinal plane, z_{01} and z_{02} denote the distances between the measurement points along the field axis, ϑ_1 and ϑ_2 are the polar angles of the arcs, and Θ_{MS} is the longitudinal scattering angle.

space points, which we assume to be measured without uncertainties, do however only provide a total of nine coordinates; additional constraints are thus needed to obtain the track parameters and scattering angles. These constraints can be obtained from MS theory since the scattering angles depend statistically on the particle type and momentum, and the material of the detector.

Starting from a hit triplet, see Fig. 1, a trajectory consisting of two arcs connecting the three-dimensional space points is constructed. It is assumed that the middle point x_1 lies in a scattering plane which deflects the particle and thus creates a kink in the trajectory. The corresponding scattering angles in the transverse and longitudinal plane are denoted by Φ_{MS} and Θ_{MS} respectively.

We assume that the particle momentum (and thus its three-dimensional radius R_{3D}) is conserved.[3] The scattering angles Φ_{MS} and Θ_{MS} have a mean of zero and variances $\sigma_\vartheta^2 = \sigma_{MS}^2$ and $\sigma_\phi^2 = \sigma_{MS}^2/\sin^2\vartheta$, which can be calculated from MS theory, using e.g. the Highland approximation [10]. The task is thus to find a unique R_{3D} which minimizes the scattering angles, explicitly the following χ^2 function:

$$\chi^2(R_{3D}) = \frac{\Phi_{MS}(R_{3D})^2}{\sigma_\phi^2} + \frac{\Theta_{MS}(R_{3D})^2}{\sigma_\vartheta^2}. \tag{1}$$

For weak MS effects the momentum dependence of the scattering uncertainty is negligible; the case of large MS effects is discussed in more detail in Section 2.4. Assuming $\frac{d\sigma_{MS}}{dR_{3D}} = 0$, the minimization of $\chi^2(R_{3D})$ is thus equivalent to solving the equation

$$\sin^2\vartheta\,\frac{d\Phi_{MS}(R_{3D})}{dR_{3D}}\,\Phi_{MS}(R_{3D}) + \frac{d\Theta_{MS}(R_{3D})}{dR_{3D}}\Theta_{MS}(R_{3D}) = 0 \tag{2}$$

for R_{3D}. The scattering angle in the transverse plane Φ_{MS} is given by

$$\Phi_{MS} = (\varphi_{12} - \varphi_{01}) - \frac{\Phi_1(R_{3D}) + \Phi_2(R_{3D})}{2} \tag{3}$$

where the bending angles Φ_1 and Φ_2 are the solutions of the transcendent equations

$$\sin^2\frac{\Phi_1}{2} = \frac{d_{01}^2}{4R_{3D}^2} + \frac{z_{01}^2}{R_{3D}^2}\frac{\sin^2\frac{\Phi_1}{2}}{\Phi_1^2},\quad \sin^2\frac{\Phi_2}{2} = \frac{d_{12}^2}{4R_{3D}^2} + \frac{z_{12}^2}{R_{3D}^2}\frac{\sin^2\frac{\Phi_2}{2}}{\Phi_2^2}. \tag{4}$$

These equations have several solutions depending on the number of half-turns of the track. However, for most practical cases it is sufficient to consider the first two solutions.

Similarly, the scattering angle in the longitudinal plane is given by

$$\Theta_{MS} = \vartheta_2 - \vartheta_1 \tag{5}$$

where the polar angles ϑ_1 and ϑ_2 can be calculated from the azimuthal bending angles using the relations

$$\sin\vartheta_1 = \frac{d_{01}}{2R_{3D}}\,\mathrm{cosec}\left(\frac{z_{01}}{2R_{3D}\cos\vartheta_1}\right),\quad \sin\vartheta_2 = \frac{d_{12}}{2R_{3D}}\,\mathrm{cosec}\left(\frac{z_{12}}{2R_{3D}\cos\vartheta_2}\right). \tag{6}$$

Alternatively the relations

$$\Phi_1 = \frac{z_{01}}{R_{3D}\cos\vartheta_1},\quad \Phi_2 = \frac{z_{12}}{R_{3D}\cos\vartheta_2} \tag{7}$$

between the azimuthal bending angles and the polar angles can be exploited.

Eqs. (4) and (6) have no algebraic solutions; they can either be solved by numerical iteration or by using a linearization around an approximate solution; the second approach is discussed in the following.

2.1. Taylor expansion around the circle solution

The circle solution describes the case of constant curvature in the plane transverse to the magnetic field $r_1 = r_2$ and no scattering in that plane, $\Phi_{MS} = 0$. This solution exists for any hit triplet and is thus a good starting point for the linearization. The radius R_C of the circle in the transverse plane going through three points is given by

$$R_C = \frac{d_{01}\,d_{12}\,d_{02}}{2\,[(\mathbf{x}_1 - \mathbf{x}_0)\times(\mathbf{x}_2 - \mathbf{x}_1)]_z}, \tag{8}$$

where d_{ij} is the transverse distance between the hits i and j of the triplet, see Fig. 1.

The bending angles for the circle solution are

$$\Phi_{1C} = 2\arcsin\frac{d_{01}}{2R_C},\quad \Phi_{2C} = 2\arcsin\frac{d_{12}}{2R_C}. \tag{9}$$

Note that the above equations have in general two solutions ($\Phi_{iC} < \pi$ and $\Phi_{iC} > \pi$) and care is needed to select the physical one, especially for highly bent tracks. The corresponding three-dimensional radii of the arcs are calculated as

$$R_{3D,1C}^2 = R_C^2 + \frac{z_{01}^2}{\Phi_{1C}^2},\quad R_{3D,2C}^2 = R_C^2 + \frac{z_{12}^2}{\Phi_{2C}^2}. \tag{10}$$

In general $\Theta_{MS} \neq 0$ such that the two radii are not identical. Using Eq. (7), polar angles for the circle solution are obtained:

$$\vartheta_{1C} = \arccos\frac{z_{01}}{\Phi_{1C}R_{3D,1C}},\quad \vartheta_{2C} = \arccos\frac{z_{12}}{\Phi_{2C}R_{3D,2C}}. \tag{11}$$

Starting from this special circle solution with no scattering in the transverse plane, we calculate the general solution $\Phi_{MS} \neq 0$ which minimizes Eq. (1) and for which momentum conservation is fulfilled, i.e. R_{3D} does not change between the segments. With the positions of the three hits given, the arc lengths and the polar angles depend only on the radius, i.e. $\Phi_{1,2} = \Phi_{1,2}(R_{3D})$ and $\vartheta_{1,2} = \vartheta_{1,2}(R_{3D})$ Eqs. (4) and (6). We can therefore perform a Taylor expansion to first order around the

[3] Energy loss due to ionization is usually small and can be either neglected or corrected for.