Nuclear Instruments and Methods in Physics Research A 843 (2017) 46–49

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Improvement in momentum resolution of parent particles using mass constraint in the rest frame

Ahmet Bingül

Department of Engineering Physics, Gaziantep University, Gaziantep 27310, Turkey

ARTICLE INFO

Keywords:
Mass constraint
Center of mass
Fast constraint
Kinematic fit

ABSTRACT

In particle physics, uncertainties in the reconstructed momentum of parent particles are introduced due to detector resolution. Traditionally, the momentum resolution of the parent particle is improved by minimizing a non-linear chi-square function via iterative methods. In this study, it is shown that the same chi-square minimization procedure results in a set of linear equations which can be solved non-iteratively in the center of mass frame of the parent particle. By using ALEPH full simulation data, the performance of the new method is compared with relatively slower iterative method for several decay channels. No significant difference between them is obtained in terms of improvement in momentum resolution. However, the new approach is found to be simple to implement and faster than that of traditional iterative method.

1. Introduction

In particle physics, a short-lived parent particle is reconstructed from its decay products. Even if the intrinsic width of the parent is very small, the invariant mass distribution of the products is smeared around its nominal mass, instead of resulting in a single spike, due to the limited momentum and the angular resolution of the particle detector used.

In order to improve the momentum and the mass resolution of the parent particle, a kinematic fit is applied to the reconstructed daughter particles. The fit is performed by varying the momenta of the daughters such that their invariant mass gives the nominal mass of the parent particle. To do that, a χ^2 function, involving the momenta of the daughters and the detector resolutions, is defined as a merit function. By minimizing the χ^2 function one can obtain the refitted values of the momentum vector of the daughters. However, the traditional χ^2 method is an iterative and a time consuming procedure since the minimization steps involves many non-linear operations and the numerical calculations of the second derivatives. Therefore, the iterative method can provide a significant CPU overhead on platforms where the speed in an issue. More detailed descriptions of the iterative methods can be found in [1–3].

On the other hand, if the χ^2 function is re-defined in the rest frame of the parent particle and the angular resolution of the detector is ignored, then the minimization procedure results in a set of linear equations which can be solved non-iteratively. Using ALEPH full simulation data [4], the iterative and the non-iterative methods are compared for two-body and three-body decays. Insignificant difference between them is observed in terms of improvement in the momentum resolution of the parent particle. However, the new approach is found to be simple to implement and faster than the iterative χ^2 method.

The event selection and the detector simulation used in the analysis are given in Section 2. The iterative method used in this study is briefly described in Section 3. The details of the new approach can be found in Section 4. The performances of the methods are presented in Section 5. Finally, a conclusion is given in Section 6.

2. Event and detector simulation

In the event simulation, the decays $\pi^0 \to \gamma\gamma$, $\eta^0 \to \gamma\gamma$, $\eta \to \pi^+\pi^-\gamma D^0 \to K\pi$, $D^\pm \to K^\mp\pi^\pm\pi^\pm$ and $D^{*0} \to D^0\pi^0$ are selected in the momentum range between 1 and 35 GeV from approximately five million e^-e^+ collision events representing simulations of hadronic Z decays at the LEP collider. The selected events are passed through the full simulation and the reconstruction program for the ALEPH detector [5] and so provide a realistic simulation of daughter momentum resolutions.

ALEPH ECAL parameterizations available for the unconverted photon energy and the angular resolution are:

$$\sigma_E/E = 0.18/\sqrt{E} + 0.009/E \quad (1)$$

$$\sigma_{\theta,\phi} = (2.5/\sqrt{E} + 0.25)\,\text{mrad} \quad (2)$$

where the photon energy, E, is in GeV.

For the charged tracks the momentum resolution is approximated by:

E-mail address: Ahmet.Bingul@cern.ch.

http://dx.doi.org/10.1016/j.nima.2016.10.060
Received 7 June 2016; Received in revised form 9 August 2016; Accepted 31 October 2016
Available online 05 November 2016
0168-9002/

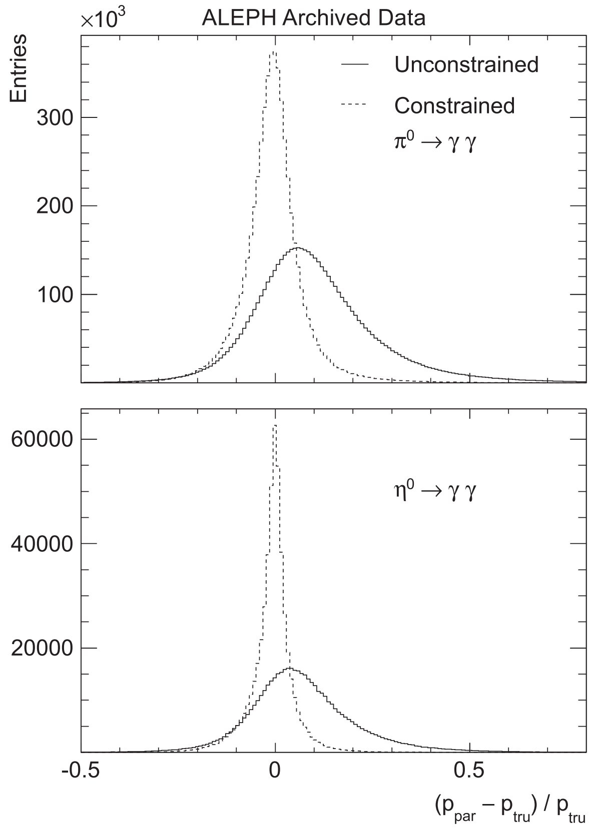

Fig. 1. $(p_{par} - p_{tru})/p_{tru}$ distributions for the unconstrained (solid line) and constrained decays (dashed line) of $\pi^0 \to \gamma\gamma$ and $\eta^0 \to \gamma\gamma$ for the momentum range between 1 and 35 GeV.

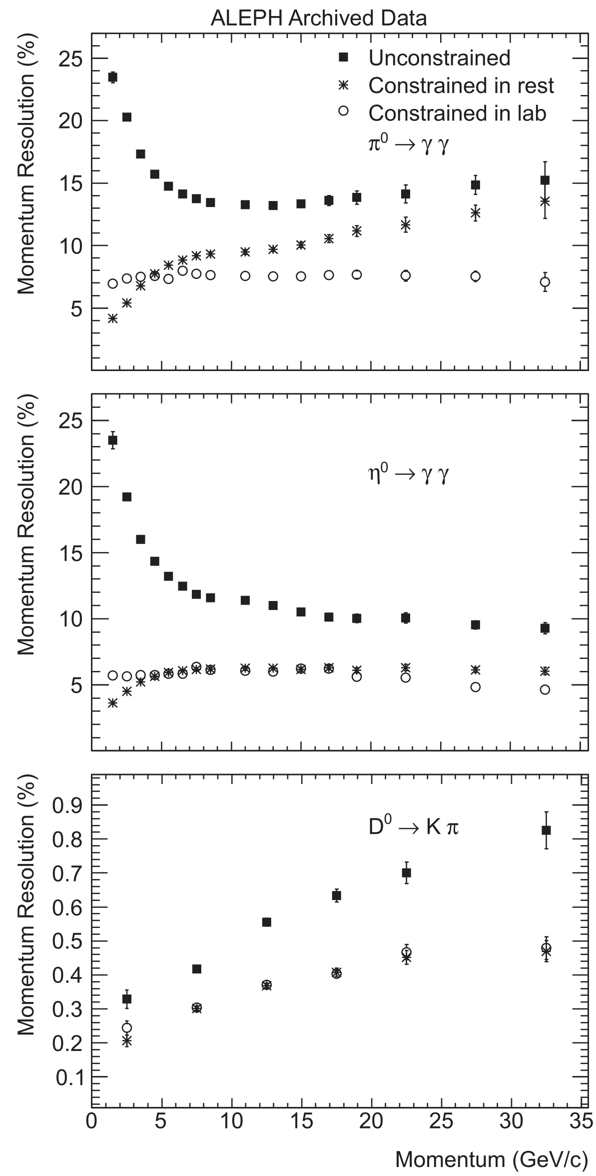

Fig. 2. Momentum resolution of $\pi^0 \to \gamma\gamma$, $\eta^0 \to \gamma\gamma$ and $D^0 \to K\pi$ decays as a function of the reconstructed parent momentum for uncontrained particles (full squares) and the constrained particles in laboratory frame (circles) and in the rest frame (stars).

$$\sigma_{p_T}/p_T = 8 \times 10^{-4} p_T \tag{3}$$

Here the transverse momentum, p_T, is in GeV. The angular resolution of the charged tracks may be ignored since the position resolution of the tracking detectors are in the order of 5 − 10 μm.

In the analysis, the unconverted photons with energy $E > 0.8$ GeV are selected. The reconstructed charged particles are required to have the pseudo-rapidity $|\eta| < 2.5$ and $p_T > 0.5$ GeV.

3. Chi-square method

By minimizing the following multi-variate chi-square function, one can obtain the refitted values of daughter's momenta:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{f_i - p_i}{\sigma_i} \right)^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{K_{ij} - \cos\theta_{ij}}{\sigma_{\cos\theta_{ij}}} \right)^2 \tag{4}$$

where n is the number of decay products, f_i and p_i are the magnitude of the refitted and the reconstructed momenta of the ith decay product respectively and σ_i is the corresponding momentum resolution of the detector. $\cos\theta_{ij}$ is the reconstructed opening angle between the products i and j, K_{ij} is the refitted value of $\cos\theta_{ij}$ and $\sigma_{\cos\theta_{ij}}$ is the resolution of $\cos\theta_{ij}$.

The first sum in the Eq. (4) represents the contribution of the momentum resolution to the total fit, while the second one stands for the angular resolutions. This is the most simple merit function for the solution. More sophisticated approaches can be found in [1,2].

The refitting parameters are not entirely free since the model has to fulfill a mass constraint. If the parent particle has the nominal mass M, then the invariant mass of the decay products must satisfy the following relation:

$$M^2 = \left(\sum_{i=1}^{n} \sqrt{f_i^2 + m_i^2} \right)^2 - \left(\sum_{i=1}^{n} \mathbf{f}_i \right)^2 \tag{5}$$

where m_i is the mass and $\mathbf{f}_i$ is the refitted momentum vector of the ith decay product respectively. Using Eq. (5), one of the refitting parameters can be eliminated.

In this study, the minimization is carried out by the Newtonian Method of the following form:

$$\mathbf{x}_{l+1} = \mathbf{x}_l - \mathbf{H}_l^{-1} \nabla \chi_l^2 \tag{6}$$

where $\mathbf{x}$ is the vector of free parameters in the fit,[1] $\mathbf{H}$ is the Hessian Matrix involving the second order partial derivatives and l is the iteration counter. If $\mathbf{x}$ is near the solution, then each iteration converges quadratically on the solution. Usually, a few iterations are required to obtain a minimum χ^2 value, however, each iteration

[1] For an n-body decay, the size of $\mathbf{x}$ is $n(n+1)/2 - 1$.