Nuclear Instruments and Methods in Physics Research A 854 (2017) 149–155

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Multipole-field measurements by sampling oblong apertures of accelerator magnets

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ARTICLE INFO

Keywords:
Magnetic Measurements
Rotating Search Coil
Multipoles
Field Harmonics
Feed-Down
Least Square
Error Analysis

ABSTRACT

The rotating search coil is a commonly used tool to measure magnetic fields of accelerator magnets. The coil intercepts the magnetic flux at a radius given by the dimensions of the measurement shaft that comprises a set of search coils for the analog bucking of the main signal from the dipole field component. For magnets of a rectangular aperture with large aspect ratio (> 3: 1) the cylindrical domain covers only a portion of the magnet bore. As the field representation is dominated by measurement errors outside that cylindrical domain, a sampling technique is required. The method presented in this paper trades the precision in the measurements against the precision in the shaft positioning and arrives at a multipole representation that covers the entire bore of the magnet.

1. Introduction

Magnetic field measurements are of fundamental importance in every accelerator project. Knowing the generated magnetic field allows to verify the magnet design (modeling errors), the magnet manufacture (magnet to magnet reproducibility) and the deviations from the desired field (input for beam physics simulations and machine operation). Furthermore, magnetic measurements are essential to study dynamic effects, such as combinations of iron hysteresis and 3D eddy-currents, which are still a challenge in magnetic field simulation.

The magnetic field of accelerator magnets is commonly described by its main field component and the higher-order field errors. In a mathematical sense, this representation is an analytic function with so-called multipole coefficients [1]. These coefficients are determined by a Fourier series expansion of one integrated field component along the design trajectory. This representation is therefore strictly two-dimensional as the magnetic field is integrated along the search coil that covers the magnet extremities and the fringe-field region.

Being the most precise tool to measure the field errors, the rotating search coil [3] nevertheless lacks in versatility, because for a highest precision, the shaft must be as large as possible. This is a problem for magnets with rectangular apertures of large aspect ratio. For rectangular apertures large, stationary fluxmeters are often used for fast ramped magnets. For static operation a combination of field measurements at different transverse positions has found attention in the measurement community.

In synchrotrons the large aspect ratio aperture is usually filled by the non-symmetric transverse beam profile which is generated by the multi-turn injection process in the horizontal plane [2]. The field representation therefore has to cover the full domain.

A field representation in elliptical coordinates is an approach that requires the field distribution on an elliptical boundary from which elliptical multipole coefficients can be extracted [4]. However, the noise spectrum on the measurement is not respected because all multipole orders must be used to express the magnetic field distribution [5]. Experience has shown that the main field component is about two orders of magnitude less precise [6] than the higher-order multipoles.

The novelty of the proposed method lies in the exclusive use of the higher-order multipoles, which allows not only to preserve but even to increase the precision of the combined result with respect to the single measurement. The computed results are valid in the entire sampled domain.

The method was first presented for magnet with round apertures in [7]. In this paper, we present the combination of three rotating-coil measurements on the mid-plane of a normal-conducting dipole magnet.

2. Fundamentals

Consider three multipole measurements acquired by a rotating search coil, as shown in (Fig. 1). Each measurement yields a set of Fourier coefficients/multipoles, which completely determine the field

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http://dx.doi.org/10.1016/j.nima.2017.01.001
Received 27 July 2016; Received in revised form 11 December 2016; Accepted 2 January 2017
Available online 03 January 2017

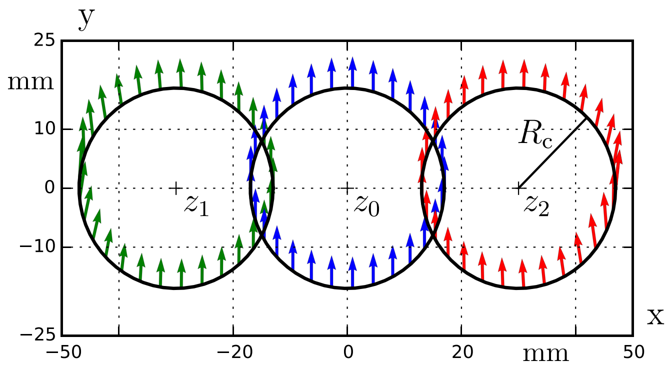

Fig. 1. Three rotating coil measurements at the positions z_i.

inside the coil radius R_c. Outside that radius, the measurement errors strongly affect the accuracy of the field representation. In order to obtain a single field representation for the entire domain (aperture of the magnet), the three measurements must be combined in a post-processing step.

2.1. Magnetic field representation

The magnetic field representation reads in complex notation [1]

$$B(z) = B_y + iB_y = \sum_{n=1} C_n \left(\frac{z}{R_c}\right)^{n-1}, \tag{1}$$

where $z = x + iy$ and the complex multipoles are given by $C_n = B_n + iA_n$. The radius R_c is the search-coil radius or may be regarded as the reference radius to which the multipoles are scaled. The scaling law $C_n(r_0) = C_n(R_c)(r_0/R_c)^{n-1}$ allows to express the multipoles at a different radius r_0. The reference radius r_0 is often noted as R_0 or R_{ref}; in this work the reference radius equals the coil radius and will consequently be denoted R_c. In measurement practice the field harmonics are determined by a rotating coil measurement, which yields the radial component of the magnetic field on a circle of the radius of the coil. Owing to the regularity of the magnetic field in an aperture free of magnetic material and current sources, the governing Laplace equation and the eigensolutions of a boundary value problem, yield a field representation inside the measurement domain. In other words, the field harmonics C_n can be determined from the boundary data established by the search-coil measurement.

2.2. Analytic continuation

Taking the complex representation of the magnetic field in Eq. (1), we can calculate the effect on the multipole-field errors by translating the reference frame into the positions of the measurement coil, $z \to z'$, $z' := z - z_i$. As this displacement stays within the bore of the magnet, free of magnetic material and current sources, the path between z and z' remains zero-homotopic as required by the method of analytic continuation. For the magnetic flux density being invariant with respect to the frame change we obtain

$$\sum_{n=1}^{\infty} C_n(z_0)\left(\frac{z}{R_c}\right)^{n-1} = B(z) = \sum_{n=1}^{\infty} C'_n(z_i)\left(\frac{z'}{R_c}\right)^{n-1}. \tag{2}$$

z_i are the position of the displaced measurements. Using the binomial series expansion for the term $(z' + z_i)^{n-1}$, the left-hand side of Eq. (2) can be transformed as follows:

$$\begin{aligned}\sum_{n=1}^{\infty} C_n(z_0)\left(\frac{z}{R_c}\right)^{n-1} &= \sum_{n=1}^{\infty} C_n(z_0)\left(\frac{z' + z_i}{R_c}\right)^{n-1}\\ &= \sum_{n=1}^{\infty}\sum_{k=1}^{n} C_n(z_0)\binom{n-1}{k-1}\left(\frac{z'}{R_c}\right)^{k-1}\left(\frac{z_i}{R_c}\right)^{n-k}\end{aligned} \tag{3}$$

Rearranging the double sum [8] according to $\sum_{n=1}^{\infty}\sum_{k=1}^{n} a_{nk} = \sum_{k=1}^{\infty}\sum_{n=k}^{\infty} a_{nk} = \sum_{n=1}^{\infty}\sum_{k=n}^{\infty} a_{kn}$ yields:

$$\sum_{n=1}^{\infty}\sum_{k=n}^{\infty} C_k(z_0)\binom{k-1}{n-1}\left(\frac{z'}{R_c}\right)^{n-1}\left(\frac{z_i}{R_c}\right)^{k-n} = \sum_{n=1}^{\infty} C'_n(z_i)\left(\frac{z'}{R_c}\right)^{n-1} \tag{4}$$

Comparing the coefficients and using the identity $\binom{a}{b} = \binom{a}{a-b}$ finally results in

$$C'_n(z_i) = \sum_{k=n}^{\infty} C_k(z_0)\binom{k-1}{k-n}\left(\frac{z_i}{R_c}\right)^{k-n}. \tag{5}$$

For measurement practice the series is truncated at an index k=K, usually at K=15 because of the limited signal-to-noise ratio:

$$C'_n(z_i) \approx \sum_{k=n}^{K} C_k(z_0)\binom{k-1}{k-n}\left(\frac{z_i}{R_c}\right)^{k-n}. \tag{6}$$

Every multipole measured with the displaced coil is coupled to every higher-order multipole in the reference frame. This effect is know as feed-down in the magnet-design community.

2.3. Synthesis of the field measurements

A synthesis of multiple magnetic measurements, presented in [7], relies on the link between the multipoles C'_n at the displaced positions and the multipoles C_n at the center. The feed-down formula from Eq. (6) is noted for the multipole orders $n = 2, 3, 4, \ldots, N$ for all displaced positions z_i. The multipoles to compute are the C_n and are the unknowns in an over-determined equation system. The equation system for the three measurements can be written as:

$$\begin{aligned}
C'_2(z_1) &= C_2(z_0)\binom{1}{0} + C_3(z_0)\binom{2}{1}\left(\frac{z_1}{R_c}\right)^1 + \cdots + C_K(z_0)\binom{K-1}{K-2}\left(\frac{z_1}{R_c}\right)^{K-2}\\
C'_3(z_1) &= C_3(z_0)\binom{2}{0} + C_4(z_0)\binom{3}{1}\left(\frac{z_1}{R_c}\right)^1 + \cdots + C_K(z_0)\binom{K-1}{K-3}\left(\frac{z_1}{R_c}\right)^{K-3}\\
&\vdots\\
C'_2(z_0) &= C_2(z_0)\\
C'_3(z_0) &= C_3(z_0)\\
&\vdots\\
C'_2(z_2) &= C_2(z_0)\binom{1}{0} + C_3(z_0)\binom{2}{1}\left(\frac{z_2}{R_c}\right)^1 + \cdots + C_K(z_0)\binom{K-1}{K-2}\left(\frac{z_2}{R_c}\right)^{K-2}\\
C'_3(z_2) &= C_3(z_0)\binom{2}{0} + C_4(z_0)\binom{3}{1}\left(\frac{z_2}{R_c}\right)^1 + \cdots + C_K(z_0)\binom{K-1}{K-3}\left(\frac{z_2}{R_c}\right)^{K-3}\\
&\vdots
\end{aligned}$$

The vertical dots indicate equations up to C'_N, where N is the highest multipole order extracted from each of the measurements. The central measurement at z_0 defines the position of the reference frame in which the reconstructed multipoles are computed. Here, the feed-down formula does not need to be applied so that the measured and reconstructed multipoles are equal and the equations become trivial.

2.3.1. Matrix notation

The equation system can be written in matrix notation as

$$\{C'\} = [M]\{C\}, \tag{7}$$

where the elements in $[M] \in \mathbb{C}^{3(N-1)\times(K-1)}$ are functions of the search coil radii, the shaft positions, and the binomial coefficients stemming from the analytic continuation. The vector $\{C'\} \in \mathbb{C}^{3(N-1)}$ contains the measured field harmonics; $\{C\} \in \mathbb{C}^{K-1}$ contains the multipoles in the reference frame, which are the unknown values to be computed. The column vectors are given by

$$\{C'\} = (C'_2(z_1), C'_3(z_1), \ldots, C'_N(z_1), C'_2(z_0), \ldots, C'_N(z_0), \ldots)^{\mathrm{T}}, \tag{8}$$

$$\{C\} = (C_2, C_3, \ldots, C_K)^{\mathrm{T}}. \tag{9}$$

The block matrix $[M]$ is composed of three inner matrices $[W_i]$, one for