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An improved equivalent circuit model of a four rod deflecting cavity

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ABSTRACT

In this paper we present an improved equivalent circuit model for a four rod deflecting cavity which calculates the frequencies of the first four modes of the cavity as well as the $\frac{R_T}{Q}$ for the deflecting mode. Equivalent circuit models of RF cavities give intuition and understanding about how the cavity operates and what changes can be made to modify the frequency, without the need for RF simulations, which can be time-consuming. We parameterise a generic four rod deflecting cavity into a geometry consisting of simple shapes. Equations are derived for the line impedance of the rods and the capacitance between the rods and these are used to calculate the resonant frequency of the deflecting dipole mode as well as the lower order mode and the model is benchmarked against two test cases; the CEBAF separator and the HL-LHC 4-rod LHC crab cavity. CST and the equivalent circuit model agree within 4% for both cavities with the LOM frequency and within 1% for the deflecting frequency. $\frac{R_T}{Q}$ differs between the model and CST by 37% for the CEBAF separator and 25% for the HL-LHC 4-rod crab cavity; however this is sufficient for understanding how to optimise the cavity design. The model has then been utilised to suggest a method of separating the modal frequencies in the HL-LHC crab cavity and to suggest design methodologies to optimise the cavity geometries.

1. Introduction

RF cavities operating in a dipole mode have a variety of applications in modern particle accelerators and colliders. Common uses of RF deflecting cavities are for longitudinal beam diagnostics [1], emittance exchangers [2], X-ray pulse compression [3] and crab-crossing of bunches in colliders [4,5]. In this report we shall focus on developing an equivalent circuit model for a four rod deflecting cavity (4RDC) which operates in a TEM-110 like mode. The model is compared to the design frequencies of the CEBAF RF separator cavity [6] and the proposed HL-LHC 4-rod crab cavity [7,8] which both show a good agreement between the equivalent circuit model and simulation results. Other deflecting cavity designs exist, such as the double quarter-wave crab cavity [9] and the RF dipole crab cavity [10]; which are also proposed for HL-LHC.

A 4RDC is a deflecting cavity containing four rods arranged in a plane, consisting of two parallel sections of two longitudinally opposing rods, as shown in Fig. 1. The four rods act as separate coupled quarter-wave resonators, which allows the cavity to resonate in the desired deflecting mode. However as there are four coupled resonators, there are four eigenmodes of the system due to the different permutations of the polarity of the charge on each rod. The eigenmodes are two dipole modes where transversely opposite rods have opposite charges giving a transverse field, and two monopole modes where the transversely opposing rods have the same polarity giving a longitudinal field.

Equivalent circuit models of RF cavities are a useful means of estimating cavity parameters such as the resonant frequency and R/Q and give intuition and understanding about how the cavity operates and what changes can be made to modify the frequency, without the need for RF simulations which can be time-consuming. Existing equivalent circuit models for 4RDCs, such as the model outlined in [11], are based on simplifications such as neglecting the capacitance in the gap between longitudinally opposed rods and ignoring the effect of the outer can of the cavity. Ignoring the end capacitance yields the same resonant frequency for all four eigenmodes, which is incorrect. Having the lower order mode (LOM) and deflecting dipole mode at the same frequency would make it difficult to damp the unwanted monopole mode hence it is beneficial to separate the two modes in frequency. An improved equivalent circuit model would provide an understanding of how best to separate these modes.

In this report, we present an improved equivalent circuit model whereby we derive equations for the line impedance of the rods for the deflecting mode as well as the LOM. We then provide a model for the capacitance between longitudinally opposed rods, starting with a physical model and then add some empirical correction terms to create a model which fits the observed results from CST simulations [12]. We then use the line impedance and end capacitance models to determine the resonant frequencies of a 4RDC for both the LOM and deflecting

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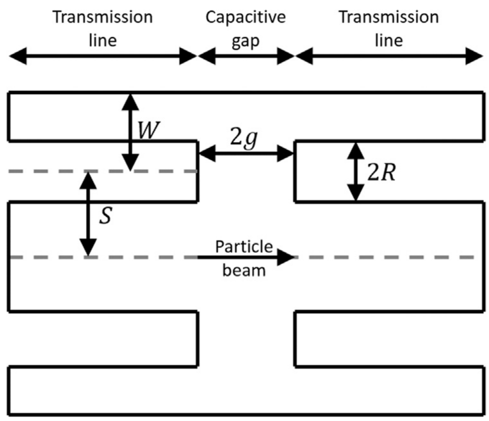

Fig. 1. A cross-sectional diagram of a generic four rod deflecting cavity.

mode. We then use this equivalent circuit model to develop a model for the $\frac{R_T}{Q}$, also known as the geometric transverse shunt impedance, for the deflecting mode of the cavity. Finally, we discuss how to use the equivalent circuit model to optimise the cavity design parameters.

2. Equivalent circuit model

From the cross-section of a generic 4RDC (Fig. 1), we may consider the cavity as consisting of three distinct regions; two transmission lines separated by a capacitive region. The geometry of the cavity has been parameterised in terms of the transverse rod separation, $2S$, the longitudinal rod separation, $2g$, the rod to wall separation, W, the rod length, L_{rod} and the rod radius, R.

We assume that the transmission line has a characteristic impedance, Z_0, and that this is terminated through the capacitive region with a load impedance, $Z_L = -\frac{j}{\omega C_{end}}$, where C_{end} is the capacitance between the end of the rod and the symmetry plane between longitudinally opposing rods (Fig. 3), which is a ground plane. This can be expressed by the equivalent circuit shown in Fig. 2 where the input impedance, Z_{in}, can be expressed as [13]

$$Z_{in} = \frac{Z_L + jZ_0 \tan\left(\frac{\omega L_{rod}}{c}\right)}{Z_0 + jZ_L \tan\left(\frac{\omega L_{rod}}{c}\right)}. \tag{1}$$

At the resonant frequency, the input impedance tends to zero, because the end is shorted, therefore from Eq. (1), the resonant frequency can be determined by solving [14]

$$\frac{1}{\omega C_{end}} = Z_0 \tan\left(\frac{\omega L_{rod}}{c}\right). \tag{2}$$

Hence if we can determine the characteristic line impedance of the relevant mode, Z_0 and the capacitance between longitudinally opposing rods, C_{end}, we can calculate the resonant frequencies of the deflecting cavity. Conversely, if we neglect the capacitance between longitudinally opposed rods, as in other equivalent circuit models of 4RDCs, from Eq. (2) we obtain $L_{rod} = \frac{\lambda}{4}$ and the frequency of all four eigenmodes becomes $\frac{c}{4L_{rod}}$.

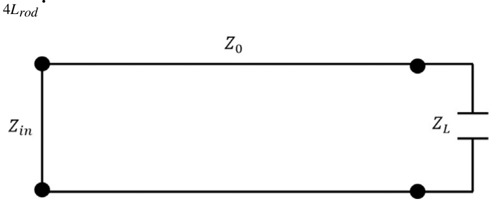

Fig. 2. An equivalent circuit for a rod in a four rod deflecting cavity described as a transmission line terminated by a capacitance.

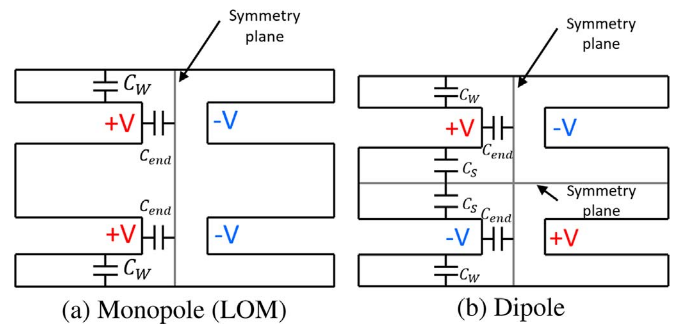

Fig. 3. Diagrams illustrating the rod potential configurations for the LOM and deflecting modes respectively. (a) Monopole (LOM) (b) Dipole.

2.1. Transmission line characteristic line impedance

The configuration of rod potentials differs between the deflecting mode and the LOM for the deflecting cavity (Fig. 3). The potential of the outer can and the symmetry planes can be considered as ground. In the LOM, the capacitances of the transversely opposing rods can be considered in parallel, thus the capacitance of the transmission line can be defined as $C_{line}^{LOM} = 2C_W$. For the deflecting mode, the capacitances of the transversely opposing rods can be considered as being in series because the rods have opposite charges; hence the capacitance of the transmission line can be defined as $C_{line}^{dipole} = \frac{1}{2}(C_W + C_S)$.

In Fig. 4, one can see the other monopole and dipole modes. As the longitudinally opposing rods have the same potential, there is no end capacitance in this system, however there is a small capacitance to ground between the end of the rod and the wall and transversely opposing rod (only for the dipole mode). However this capacitance is small compared to the line impedance of the rod and therefore the difference in frequency between the two eigenmodes is small. If g tends to zero, the frequency of both eigenmodes tends to $\frac{c}{4L_{rod}}$. Furthermore these eigenmodes have very low shunt impedance compared to the other modes and can therefore be neglected.

In this paper, we consider the case where the outer can of the cavity has a rectangular cross section, such that the walls parallel to the deflecting plane are far away from the rods. This allows us to neglect image charges out of the deflecting plane as these can significantly alter the line impedance of the rods, particularly for the LOM. Image charges out of the deflecting plane can be extremely difficult to model as the distance to image charges tend to be over a continuous range rather than at discrete values, hence the perturbation to the line impedance due to these image charges cannot be expressed as a series or product expansion.

In order to determine the line impedance of the rods, we shall consider each rod as a uniform line charge, where the total charge of each rod is $q_{rod} = \pm\lambda_{rod}L_{rod}$. The potential at a transverse distance, x, from a uniform line charge of length L_{rod} can be expressed as

$$V = \frac{\lambda_{rod}}{4\pi\epsilon_0} \ln\left(\frac{\sqrt{x^2 + L_{rod}^2} + L_{rod}}{\sqrt{x^2 + L_{rod}^2} - L_{rod}}\right). \tag{3}$$

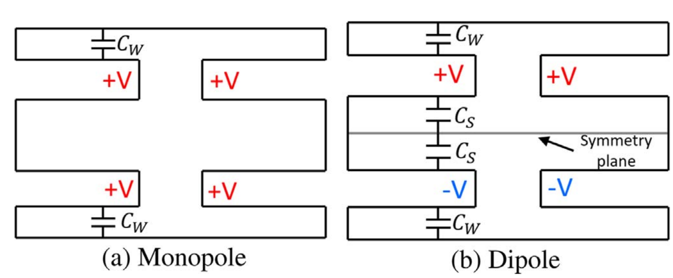

Fig. 4. Diagrams illustrating the rod potential configurations for the other monopole and dipole eigenmodes. (a) Monopole (b) Dipole.