Available online at www.sciencedirect.com

ScienceDirect

Nuclear and Particle Physics Proceedings 285–286 (2017) 14–19

www.elsevier.com/locate/nppp

A bottom up determination of lepton mass matrices

Andrea Romanino

SISSA/ISAS and INFN, I–34136 Trieste, Italy

Abstract

We illustrate a bottom up approach to the problem of understanding the origin of lepton flavour based on a simple "stability" principle, according to which physical flavour observables should be stable with respect to small variations of individual matrix elements.

Keywords:
Models of neutrino masses and mixings

1. Introduction

Understanding the origin of the pattern of fermion masses and mixings remains one of the most fascinating issues in the physics of fundamental interactions. On the other hand, the hopes to find clues from the comparison of theoretical models and experimental data have to confront the limited number of data available and the landscape of models, with its dense set of predictions. It is therefore worth attempting to move from the usual perspectives (the one based on symmetries, for example) and find a relatively model-independent angle on the problem.

Here I will descrive a bottom-up approach based on a single, simple, credible hypothesis, the "stability" principle. As will see, the latter allows, in certain cases, to gather a plethora of model-independent information on the lepton mass matrices [1, 2].

In the standard approach, a model of flavour is proposed, based for example on a symmetry principle; the lepton mass matrices are deduced from the model; the physical observables, masses and mixings, are then recovered by diagonalising the mass matrices. In our approach, on the contrary, the precisely known values of the physical observables are used to infer, in a relatively model-independent way, information on the form of the lepton mass matrices; which can then provide direct information on the physics generating them.

In the Standard Model (SM) the mass matrices are not physical objects, as they depend on the choice of a basis in flavour space. Here, we will assume the existence of a privileged basis in flavour space, determined by the physics ultimately responsible of the form of the mass matrices [1]. This is the basis in which the entries of the lepton mass matrices are directly related to the fundamental parameters of the theory, and is the basis in which we will write the mass matrices.

2. The stability principle

The tool that allows us to infer information on the form of the mass matrices is what we call the "stability principle". According to such a principle, small physical quantities (such as m_e/m_τ, m_μ/m_τ, $|\Delta m^2_{12}/\Delta m^2_{23}|$) should be stable with respect to small variations of individual matrix entries.[2]

The motivation of such an assumption is clear: an understanding of the smallness of the electron mass, for example, requires its smallness not to be accidental, i.e. requires its stability with respect to variations

[1] This is indeed what happens in the case of flavour symmetries, because the definition of the symmetry involves specifying a basis, or because its spontaneous breaking does.

[2] See also [3, 4, 5, 6, 7] for alternative approaches to natural mass matrices.

http://dx.doi.org/10.1016/j.nuclphysbps.2017.03.004

of independent, fundamental parameters. The only non-trivial assumption we are making is therefore that matrix elements correspond to independent fundamental parameters. Because of that, our assumption is not fully general, as correlations among different matrix entries might actually arise, e.g. because of non-abelian symmetries, but it can easily extended to the general case. Moreover, the case we will consider here, in which different matrix entries are considered to be independent, is not less motivated, as experimental hints could have piled up by now in favour of models predicting correlations, but they have not so far.

The mathematical formulation of the stability principle is the following. The size (with respect to the larger scales $O(m_\tau)$ involved in the charged lepton mass matrix) of the mass of a light charged fermion m_k, $k = 1, 2$, is stable with respect to small but finite variations of the entry M^E_{ij} of the charged lepton mass matrix iff

$$\frac{\Delta m_k}{m_k} \lesssim \left|\frac{\Delta M^E_{ij}}{M^E_{ij}}\right| \quad \text{when} \quad |\Delta M^E_{ij}| \ll |M^E_{ij}|, \tag{1}$$

where Δm_k is the variation of m_k caused by the variation of M^E_{ij}. Analogously, the smallness of the solar squared mass difference (with respect to the atmospheric one) is stable with respect to small but finite variations of the entry M^ν_{ij} of the neutrino mass matrix iff

$$\frac{\Delta(\Delta m^2_{12})}{\Delta m^2_{12}} \lesssim \left|\frac{\Delta M^\nu_{ij}}{M^\nu_{ij}}\right| \quad \text{when} \quad |\Delta M^\nu_{ij}| \ll |M^\nu_{ij}|, \tag{2}$$

where $\Delta(\Delta m^2_{12})$ is the variation of Δm^2_{12} caused by the variation of M^ν_{ij}. The stability requirement is reminiscent of the requirement of the absence of fine-tuning [8], but it uses finite (not infinitesimal) differences. Using finite differences is important here because the infinitesimal variation can miss instabilities that arise only when the variation is small, but finite and larger than e.g. m_e/m_τ (in the case of the stability of m_e).

3. Stable neutrino mass matrices

Interestingly enough, the stability requirement is sufficient to get non-trivial information on the structure of the lepton mass matrices. We refer to [1, 2] for the technical details. Here, it suffices to say that the requirement translates, at the leading order in an expansion in the small quantities assumed to be stable, into simple algebraic conditions on the matrix entries.

Let us illustrate the implications for the mass matrices, starting with the neutrino one.

The first result is that the neutrino mass matrix can only have one of the four forms in Tab. 1 in the limit $\Delta m^2_{12}/\Delta m^2_{23} \to 0$, up to a permutation of the rows and columns. The four textures are well known, but they have never been rigorously associated to stability, nor obtained as the solution of simple algebraic conditions. The pattern of neutrino masses associated to each of the four textures is also shown. Interestingly, future measurements might lead to the identification of the neutrino texture. For example, if the sum of neutrino masses turned out to be out of reach and the determination of the sign of Δm^2_{23} pointed at a normal ordering, that would select texture D. If the sum of neutrino masses will be in the range accessible by future experiments, this will force a semi-degenerate spectrum, and will select texture A. The latter is a particularly interesting possibility that we now discuss in greater detail.

4. The semi-degenerate case

We say that the light neutrino mass spectrum is "semi-degenerate"[3] when the two neutrinos ν_1 and ν_2 are quasi-degenerate, and the third neutrino is neither hierarchically larger or smaller than $\nu_{1,2}$, nor degenerate. Fig. 1 shows that in a significant range below the present bound on m_tot, here taken to be $m_\text{tot} < 0.23\,\text{eV}$ [10], corresponding to the right edge of the plot, the neutrino spectrum is indeed semi-degenerate, with

$$m_1^2 \approx m_2^2 \equiv m^2, \quad m_3 \sim m, \quad \epsilon^2 \equiv \frac{\Delta m^2_{12}}{2m^2} \ll 1. \tag{3}$$

Semi-degeneracy is particularly interesting because it corresponds to a sum of light neutrino masses m_tot near the present experimental bound, and because it can be established by the measurement of the absolute neutrino mass scale. For our purposes, the case of semi-degeneracy is important also because it maximises the information on lepton mass matrices that can be extracted from the stability principle, as we now see.

Suppose that the absolute scale of neutrino masses turned out to be close to the experimental limit, thus implying a semi-degenerate neutrino spectrum; then, if the stability principle holds, the leading order structure of the neutrino mass matrix is first of all known to be in the form (A) in Tab. 1. On top of that, the stability requirement allows to get information on the size of the

[3] Sometimes called "partially degenerate" [9], although this terminology is sometimes used with different meanings.