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Nuclear and Particle Physics Proceedings 282–284 (2017) 144–148

www.elsevier.com/locate/nppp

The strong coupling from ALEPH tau decays

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Abstract

The strong coupling from ALEPH tau decays. We use the publically available non-strange spectral function from ALEPH tau decays to critically analyze the different determinations of $\alpha_s(m_\tau^2)$ that can be found in the literature and the numerical impact of their possible weaknesses. We also introduce some novel approaches. We find that perturbative uncertainties dominate. Our results with different approaches are very stable. Our final value is $\alpha_s(m_\tau^2) = 0.328 \pm 0.013$

Keywords: QCD, Strong Coupling, Tau Decays

1. Introduction

One of the most powerful tests of asymptotic freedom of QCD comes from the determination of the strong coupling from inclusive τ decays [1, 2]. In this work we summarize our recently made determination of Ref. [3].

The hadronic decay width can obtained from [4]

$$\begin{aligned} R_\tau &\equiv \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \\ &= 12\pi S_{\text{EW}} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \\ &\qquad \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \operatorname{Im}\Pi^{(1)}(s) + \operatorname{Im}\Pi^{(0)}(s)\right], \quad (1) \end{aligned}$$

where $S_{EW} = 1.0201 \pm 0.0003$ contains the renormalization-group-improved electroweak correction [5–7] and

$$\Pi^{(J)}(s) \equiv \sum_{q=d,s} |V_{uq}|^2 \left(\Pi^{(J)}_{uq,V}(s) + \Pi^{(J)}_{uq,A}(s)\right), \quad (2)$$

are the two-point correlation function of quark currents. Since one can extract the invariant-mass and spin of the final hadronic system of the τ decays, one has experimental access to the different spectral functions $\operatorname{Im}\Pi^J_{uq,J}(s)$. We make use of the most precise non-strange spectral functions available $\rho(s) = \frac{1}{\pi}\operatorname{Im}\Pi_{\mathcal{J}}(s) \equiv \frac{1}{\pi}\operatorname{Im}\Pi^{(1+0)}_{ud,\mathcal{J}}(s)$, coming from the last update of the ALEPH collaboration [8].

2. Theoretical Framework

2.1. Operator Product Expansion (OPE) of the correlators

For large-euclidean momentum, the correlators can be expanded into series of local operators weighted by their Wilson coefficients, which can be calculated using perturbative QCD [9]. At $Q^2 \sim m_\tau^2$, the numerical contribution to the different observables is dominated by the purely perturbative part.

In order to compare the theoretical OPE prediction with the experimental data, we can make use of the analytic extension of the correlator, which is well defined in all the complex the plane but in the hadronic cut in the positive real axis. Using this, it is straightforward to obtain the exact relation [4, 10, 11]

$$\begin{aligned} A^\omega_{V/A}(s_0) &\equiv \int_{s_{\text{th}}}^{s_0} \frac{ds}{s_0}\, \omega(s) \operatorname{Im}\Pi_{V/A}(s) \\ &= \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi_{V/A}(s), \quad (3) \end{aligned}$$

http://dx.doi.org/10.1016/j.nuclphysbps.2016.12.027

where $\omega(s)$ is any weight function analytic in all complex plane except in the positive real axis. We will be able to extract the experimental $A^{\omega}_{V/A}(s_0)$ from the second term of Eq. (3) and the theoretical one from the third term of the same equation using the OPE of the correlator.

2.2. Perturbative contribution

The dominant contribution to $A^{\omega}_{V/A}(s_0 \sim m_\tau^2)$ is purely perturbative. It is precisely this fact, as well as the high sensitivity of this contribution to $\alpha_s(s \sim m_\tau^2)$, which allows such a precise determination. In order to calculate the purely perturbative contribution, one can make use of the Adler function [12], known up to 4 loops [13–18]:

$$D(s) \;\equiv\; -s\,\frac{d\,\Pi^P(s)}{ds} \;=\; \frac{1}{4\pi^2}\,\sum_{n=0}\tilde{K}_n(\xi)\,a_s^n(-\xi^2 s)\,, \quad (4)$$

where $\tilde{K}_n$ are known up to $n = 4$ and $a_s(m_\tau^2) \equiv \frac{\alpha_s(m_\tau^2)}{\pi}$ satisfies the renormalization group equation

$$2\frac{s}{a_s}\frac{da_s}{ds} = \sum_{n=1}\beta_n a_s^n(s)\,. \quad (5)$$

In order to solve the integral of Eq. (3) one can either expand $A^{\omega,P}(s_0)$ in a fixed order in $\alpha_s(\xi^2 s_0)$ (FOPT) or use the exact solution to the differential equation (5) in the $\beta_{n>5} = 0$ approximation, which resums large logarithms (CIPT) [10, 19].

For a given perturbative approach, FOPT or CIPT, we cut Eq. (4) in $n = 5$ taking $K_5 = 275 \pm 400$ [20] and varying the scale dependence that arises from the fact we are cutting the series in the interval $\xi^2 = \{0.5, 2\}$ as a conservative estimates of perturbative uncertainties. The difference between FOPT and CIPT is for some moments $A^{\omega}(s_0)$ larger than these perturbative uncertainties precisely because of the large logarithms that CIPT resums. Taking this into account, and in the absence of a better understanding of higher perturbative corrections, we average the FOPT and CIPT and add quadratically half of the difference between both values to give a conservative final value.

2.3. Non-Perturbative contribution

The non-perturbative contributions due to the $D \geq 4$ operators to a given moment $A^{\omega,P}$ can be safely aproximated as functions of effective dimensional condensates O_D

$$A^{\omega,NP}_{V/A}(s_0) = \pi\sum_D a_{-1,D}\,\frac{O_{D,\,V/A}}{s_0^{D/2}}\,, \quad (6)$$

with

$$\omega(-s_0 x) \;=\; \sum_n a_{n,D}\;x^{n+D/2}\,. \quad (7)$$

The moment associated to R_τ, whose weigth function is $\omega(s_0 x) = (1 - x^2)(1 + 2x)$, is only sensitive the $D = 6$ and the $D = 8$ condensates, supressed by m_τ^6 and m_τ^8. Together with the cancellation of O_6 in the $V + A$ channel, the $D \geq 4$ contribution to R_τ happens to be very supressed [4, 21].

In addition to the non-perturbative OPE contributions, one has to take into account the differences between the physical correlators and their OPE aproximant. These differences are known as quark-hadron duality violations (DVs) [22–29]. Using Eq. (3), the contribution of duality violations to the physical observables studied are given by

$$\Delta A^{\omega,\mathrm{DV}}_{V/A}(s_0) \;\equiv\; \frac{i}{2}\oint_{|s|=s_0}\frac{ds}{s_0}\omega(s)\left\{\Pi_{V/A}(s) - \Pi^{\mathrm{OPE}}_{V/A}(s)\right\}$$
$$= -\pi\int_{s_0}^{\infty}\frac{ds}{s_0}\omega(s)\Delta\rho^{\mathrm{DV}}_{V/A}(s)\,. \quad (8)$$

These DVs are reduced using pinched weight functions [4, 11], which are functions that avoid the contribution to the integral of the region near the cut in the positive real axis, where the OPE is badly defined. Additionally, they decrease very fast with the opening of the higher multiplicity hadronic thresholds [30]. Therefore, they become very small at $s_0 \sim m_\tau^2$, specially in the more inclusive channel $V + A$. We are able to make reliable and conservative estimates of DV uncertainties looking at the stability of the strong coupling determination both by taking moments that depend on these DVs in different ways and changing s_0.

3. Results

3.1. ALEPH-like fits

First we reproduce the determination of the ALEPH collaboration [8]. They take $s_0 = m_\tau^2$ and the moments associated to the weight functions

$$\omega_{kl}(s) \;=\; \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^{l}\left(1+\frac{2s}{m_\tau^2}\right), \quad (9)$$

with $(k, l) = \{(0, 0), (1, 0), (1, 1), (1, 2), (1, 3)\}$. For these moments, duality violations are supressed by, at least, double pinching. They depend on α_s and $O_{4,6,8...16}$. The fit becomes possible when one neglects the contribution of the higher energy condensates, whose contribution is supressed by powers of m_τ^2 (Eq. 6).