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Nuclear and Particle Physics Proceedings 276–278 (2016) 189–192
www.elsevier.com/locate/nppp

Solving the Balitsky-Kovchegov equation at next to leading order accuracy

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Abstract

We solve the Balitsky-Kovchegov small-x evolution equation in coordinate space. We find that the solution to the equation is unstable when using an initial condition relevant for phenomenological applications at leading order. The problematic behavior is shown to be due to a large double logarithmic contribution. The same problem is found when the evolution of the "conformal dipole" is solved, even though the double logarithmic term is then absent from the evolution equation.

Keywords: BK, DIS, CGC

1. Introduction

The Color Glass Condensate [1] effective theory of QCD at high energy has been shown to be in good agreement with large amount of experimental data on, for example, deep inelastic scattering, single and double inclusive particle production and exclusive vector meson production, see e.g. [2, 3, 4]. There are two main ingredients in these calculations: first, one needs the small-x evolution equation such as the Balitsky-Kovchegov (BK) equation [5, 6] which describes the evolution of the dipole-target scattering amplitude as a function of energy, or equivalently, Bjorken-x. The dipole amplitude at initial Bjorken-x is the second necessary input, and it can not be obtained from perturbative calculations but it must be fit to experimental data. The fact that one can indeed obtain a good description of the precise combined HERA single inclusive data [7] has been one of the tightest tests for the CGC [3, 8].

An important next step for the description of the saturation phenomena from the CGC framework is to develop the CGC theory to the next to leading order accuracy. First steps in this direction have been taken by deriving e.g. the photon impact factor [9] and single inclusive cross section [10] at this order. The NLO BK equation is also known [11], but no solution to it existed before our work [12].

2. The BK equation at NLO

The BK evolution equation for the dipole operator S, which is a correlator of Wilson lines U such that $S(x-y) = 1/N_c\langle \operatorname{Tr} U^\dagger(x)U(y)\rangle$. At NLO accuracy the equation reads

$$\begin{aligned}\partial_y S(r) &= \frac{\alpha_s N_c}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)] \\ &+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] \\ &+ \frac{\alpha_s^2 n_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]. \end{aligned} \quad (1)$$

The quark and the antiquark of the parent dipole are at transverse positions x and y, and the daughter dipole sizes are $X = |x-z|$, $Y = |y-z|$, $X' = |x-z'|$ and $Y' = |y-z'|$, and r is the size of the parent dipole. The convolutions $\otimes$ are taken by integrating over the daughter dipole sizes (z in K_1 and both z and z' in K_2 and K_f).

The kernel K_1 includes the leading order BK kernel, the running coupling part and an α_s^2 correction, as

$$K_1 = \frac{r^2}{X^2Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2-Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c} - \ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]. \quad (2)$$

http://dx.doi.org/10.1016/j.nuclphysbps.2016.05.041

We implement the running coupling by replacing the terms proportional to the β function coefficient by the Balitsky running coupling prescription from Ref. [13]. The kernel K_1 then reads

$$\frac{\alpha_s N_c}{2\pi^2} K_1 = \frac{\alpha_s(r) N_c}{2\pi^2} \times \left[\frac{r^2}{X^2 Y^2} + \frac{1}{X^2}\left(\frac{\alpha_s(X)}{\alpha_s(Y)} - 1\right) + \frac{1}{Y^2}\left(\frac{\alpha_s(Y)}{\alpha_s(X)} - 1\right)\right] + \frac{\alpha_s(r)^2 N_c{}^2}{8\pi^3} \frac{r^2}{X^2 Y^2} \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c} - 2 \ln \frac{X^2}{r^2} \ln \frac{Y^2}{r^2} \right]. \tag{3}$$

We will later refer to the part proportional to $\ln X^2/r^2 \ln Y^2/r^2$ as the double logarithmic term.

The kernels K_2 and K_f are combinations of rational expressions of transverse separations and a logarithm $\ln X^2 Y'^2/(X'^2 Y^2)$. Note that this logarithm vanishes in the small parent dipole limit where $x \to y$, in contrast to the double logarithm. The coupling constant α_s is evaluated at the scale set by the parent dipole, as it is the only external scale. For explicit expressions, we refer the reader to Refs. [11, 12].

As an initial condition for the NLO BK equation we use a modified McLerran-Venugopalan (MV) model

$$N(r) = 1 - S(r) = 1 - \exp\left[-\frac{(r^2 Q_{s,0}^2)^\gamma}{4} \ln\left(\frac{1}{r\Lambda_{QCD}} + e\right)\right]. \tag{4}$$

Here the MV model is modified by introducing an anomalous dimension γ which controls the power-like tail of the dipole amplitude at small dipole sizes. The leading order fits to the HERA data prefer [8] values of $\gamma \sim 1.1$, which then reduces during the evolution to $\gamma \sim 0.8$. The constant $Q_{s,0}$ parametrizes the saturation scale at initial Bjorken-x. In this work, we do not seek for parameter values that are compatible with the experimental data. In practice, $Q_{s,0}$ controls the relative importance of the NLO terms as it scales the value of α_s.

3. Solution to the NLO BK

In Fig. 1 we show the evolution speed $\partial_y N(r)/N(r)$ as a function of the dipole size for the MV model ($\gamma = 1$) initial condition. At small initial saturation scales $Q_{s,0}/\Lambda_{QCD}$, when the strong coupling constant and the NLO corrections are largest, the evolution speed is negative at all dipole sizes. With smaller values of α_s (larger saturation scale) the evolution speed turns negative at small dipole sizes when $r \ll 1/Q_s$.

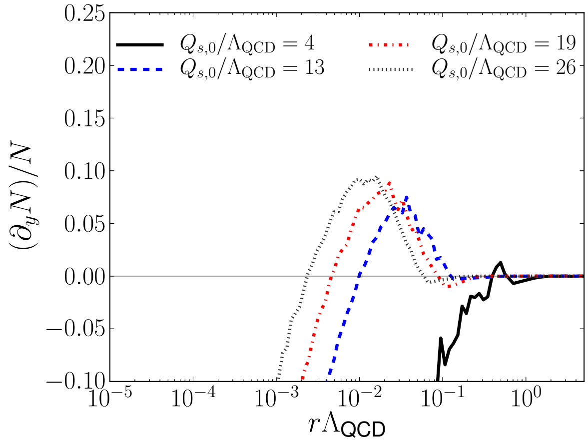

Figure 1: Evolution speed of the dipole amplitude at initial condition (MV model with $\gamma = 1$) with different values for the initial saturation scale.

The negative evolution speed is unphysical, as it corresponds to having an unintegrated gluon distribution that decreases when it is probed at smaller and smaller x. However, having $\partial_y N/N \sim \ln r$ in the small r limit is a signal of mathematical instability, as in that case there is a small (but finite) r below which the dipole amplitude becomes negative in one step dy of the rapidity evolution. On the other hand, the definition of the dipole amplitude $N(x-y) = 1 - 1/N_c \langle \mathrm{Tr}\, U^\dagger(x) U(y)\rangle$ requires that $N(r) \to 0$ in the limit $r \to 0$. Also, if the dipole amplitude does not satisfy this requirement the z integral in the leading order equation does not converge. In our numerical analysis we impose by hand a constraint $N(r) \geq 0$.

Let us then trace back the origin of the negative evolution speed. In Fig. 2 we show contributions to $\partial_y N/N$ originating from the different terms of the NLO BK equation. We observe that the double logarithmic term, which is part of the kernel K_1, is the one that drives the evolution speed negative. The other NLO corrections also decrease the evolution speed but do not cause the problematic $\partial_y N/N \sim \ln r$ behavior.

It has been argued in Ref. [14] that the double logarithmic contributions should be resummed to all orders. The resummation effectively removes the double logarithmic term from the kernel K_1 and multiplies the leading order BK kernel r^2/X^2Y^2 by an oscillatory factor, which expanded to order α_s^2 gives the double logarithmic term to the kernel K_1. The initial condition is also modified by the resummation procedure. We implement this resummation in our analysis and show in Fig. 3 the