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Exact solutions of the Boltzmann equation and optimized hydrodynamic approaches for relativistic heavy-ion collisions☆

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Abstract

Several recent results are reported from work aiming to improve the quantitative precision of relativistic viscous fluid dynamics for relativistic heavy-ion collisions. The dense matter created in such collisions expands in a highly anisotropic manner. Due to viscous effects this also renders the local momentum distribution anisotropic. Optimized hydrodynamic approaches account for these anisotropies already at leading order in a gradient expansion. Recently discovered exact solutions of the relativistic Boltzmann equation in anisotropically expanding systems provide a powerful testbed for such improved hydrodynamic approximations. We present the latest status of our quest for a formulation of relativistic viscous fluid dynamics that is optimized for applications to relativistic heavy-ion collisions.

Keywords: quark-gluon plasma, viscous fluid dynamics, Boltzmann equation, relativistic heavy-ion collisions

1. Motivation

Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions. It is an effective macroscopic description based on coarse-graining (via a gradient expansion) of the underlying microscopic dynamics. Its systematic construction is still a matter of debate, complicated by the existence of a complex hierarchy of micro- and macroscopic time scales that are not well separated in relativistic heavy-ion collisions. Exact solutions of the highly nonlinear microscopic dynamics can serve as a testbed for macroscopic hydrodynamic approximation schemes. Such solutions have been found for the Boltzmann equation in the Relaxation Time Approximation (RTA), which describes weakly interacting systems, under the assumption of highly symmetric flow patterns and density distributions (Bjorken [1] and Gubser [2] flows) of the system [3, 4, 5, 6, 7]. Here we use both to test different hydrodynamic expansion schemes.

2. Kinetic theory vs. hydrodynamics

Hydrodynamics is an effective theory whose form is independent of the microscopic interaction strength. Its equations can thus be derived from kinetic theory in a window of weak coupling and small pressure gradients where both approaches are simultaneously valid. [We will here use the RTA Boltzmann equation as our starting point.] Only the values of the transport coefficients and the equation of state depend on the microscopic coupling strength; for the strongly coupled quark-gluon plasma created in heavy-ion collisions, they must be obtained with non-perturbative methods.

The conserved macroscopic currents $j^\mu = \langle p^\mu \rangle$ (particle current) and $T^{\mu\nu} = \langle p^\mu p^\nu \rangle$ (energy-momentum

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tensor) are obtained by taking momentum moments $\langle h(p)\rangle \equiv \frac{g}{(2\pi)^3}\int \frac{d^3p}{E_p} h(p) f(x,p)$ of the distribution function $f(x,p)$ (g is the degeneracy factor.) Hydrodynamic equations are obtained by splitting the distribution function into a leading-order contribution f_0, parametrized through macroscopic observables as [8, 9, 10]

$$f_0(x,p) = f_0\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\tilde{T}(x)}\right), \tag{1}$$

and a smaller first-order correction δf ($|\delta f/f_0| \ll 1$):

$$f(x,p) = f_0(x,p) + \delta f(x,p). \tag{2}$$

In Eq. (1), $\Xi^{\mu\nu}(x) = u^\mu(x)u^\nu(x) - \Phi(x)\Delta^{\mu\nu}(x) + \xi^{\mu\nu}(x)$, where the hydrodynamic flow field $u^\mu(x)$ defines the local fluid rest frame (LRF) and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the spatial projector in the LRF. $\Phi(x)$ and the tensor $\xi(x)$ partially account for bulk viscous effects and shear-viscous deviations from local momentum isotropy in anisotropically expanding systems. $\tilde{T}(x)$, $\tilde{\mu}(x)$ are the effective temperature and chemical potential in the LRF.

$u^\mu(x)$, $\tilde{T}(x)$, and $\tilde{\mu}(x)$ are fixed by the Landau matching conditions, requiring $u^\mu u_\mu = 1$:

$$T^\mu{}_\nu u^\nu = \mathcal{E} u^\mu, \quad \langle u{\cdot}p\rangle_{\delta f} = \langle (u{\cdot}p)^2\rangle_{\delta f} = 0. \tag{3}$$

Here the eigenvalue $\mathcal{E}(\tilde{T},\tilde{\mu};\xi,\Phi)$ is the LRF energy density. The true local temperature $T(\tilde{T},\tilde{\mu};\xi,\Phi)$ and local chemical potential $\mu(\tilde{T},\tilde{\mu};\xi,\Phi)$ are introduced by demanding $\mathcal{E}(\tilde{T},\tilde{\mu};\xi,\Phi) = \mathcal{E}_{\rm eq}(T,\mu)$ and $\mathcal{N}(\tilde{T},\tilde{\mu};\xi,\Phi) \equiv \langle u{\cdot}p\rangle_{f_0} = \mathcal{R}_0(\xi,\Phi)\mathcal{N}_{\rm eq}(T,\mu)$ where $\mathcal{E}_{\rm eq}$, $\mathcal{N}_{\rm eq}$ are the thermal equilibrium energy and particle densities and $\mathcal{R}_0$ is a factor that depends on the viscous deformations ξ and Φ of the local momentum distribution [13, 9, 10].

Writing $T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu} \equiv T_0^{\mu\nu} + \Pi^{\mu\nu}$, $j^\mu = j_0^\mu + \delta j^\mu \equiv j_0^\mu + V^\mu$, the conservation laws

$$\partial_\mu T^{\mu\nu}(x) = 0, \quad \partial_\mu j^\mu(x) = \frac{\mathcal{N}(x) - \mathcal{N}_{\rm eq}(x)}{\tau_{\rm rel}(x)} \tag{4}$$

are sufficient to determine $u^\mu(x)$, $T(x)$, $\mu(x)$, but not the dissipative corrections $\xi^{\mu\nu}$, Φ, $\Pi^{\mu\nu}$, and V^μ; their evolution is controlled by microscopic physics. Different hydrodynamic approaches can be distinguished by the assumptions they make about the dissipative corrections and/or the approximations they use to derive their dynamics from the underlying Boltzmann equation:

1. Ideal hydrodynamics assumes local momentum isotropy of f, setting f_0 to be isotropic ($\xi^{\mu\nu} = 0$) and all dissipative currents to zero: $\Phi = \Pi^{\mu\nu} = V^\mu = 0$.

2. Navier-Stokes (NS) theory maintains local momentum isotropy at leading order (i.e. in f_0), sets $\Phi = 0$, and postulates instantaneous constituent relations for $\Pi^{\mu\nu}$ and V^μ by introducing viscosity and heat conduction as transport coefficients that relate these flows to their driving forces, but ignores the microscopic relaxation time that is needed for these flows to adjust to their Navier-Stokes values. This leads to acausal signal propagation.

3. Israel-Stewart (IS) theory [11] improves on NS theory by evolving $\Pi^{\mu\nu}$ and V^μ dynamically, with evolution equations derived from moments of the Boltzmann equation, keeping only terms linear in the Knudsen number $\mathrm{Kn} = \lambda_{\rm mfp}/\lambda_{\rm macro}$.

4. Denicol-Niemi-Molnar-Rischke (DNMR) theory [12] improves IS theory by keeping nonlinear terms up to order Kn^2, $\mathrm{Kn}\cdot\mathrm{Re}^{-1}$ when evolving $\Pi^{\mu\nu}$ and V^μ. (Terms of second order in the inverse Reynolds number Re^{-1} vanish in the RTA used here but would otherwise appear, too.)

5. Anisotropic hydrodynamics (aHydro) [13] allows allows for a leading-order local momentum anisotropy ($\xi^{\mu\nu}$, $\Phi \neq 0$), evolved according to equations obtained from low-order moments of the Boltzmann equation, but ignores residual dissipative flows: $\Pi^{\mu\nu} = V^\mu = 0$.

6. Viscous anisotropic hydrodynamics (vaHydro) [14] improves on aHydro by additionally evolving (using IS or DNMR theory) the residual dissipative flows $\Pi^{\mu\nu}$, V^μ generated by the deviation δf around the locally anisotropic leading-order distribution function f_0.

3. Exact solutions of the Boltzmann equation

3.1. Systems undergoing Bjorken flow

For highly symmetric flow profiles the Boltzmann equation can be solved exactly in RTA. Bjorken flow [1] describes the dynamics of a longitudinally boost invariant, transversally homogeneous system. A system with these symmetries is most conveniently discussed in Milne coordinates (τ, r, ϕ, η) where $\tau = (t^2 - z^2)^{1/2}$ and $\eta = \frac{1}{2}\ln[(t-z)/(t+z)]$. Bjorken flow is static in these coordinates, $u^\mu = (1,0,0,0)$. In Cartesian coordinates this implies a longitudinal flow velocity profile $v_z = z/t$ [1]. The metric in Milne coordinates is $ds^2 = d\tau^2 - dr^2 - r^2 d\phi^2 - \tau^2 d\eta^2$. The Bjorken symmetry restricts the possible dependence of the distribution function $f(x,p)$ to $f(x,p) = f(\tau; p_\perp, w)$ where $w = tp_z - zE = \tau m_\perp \sinh(y-\eta)$ is boost invariant. The RTA Boltzmann equation then simplifies to an ordinary differential equation

$$\partial_\tau f(\tau; p_\perp, w) = -\frac{f(\tau; p_\perp, w) - f_{\rm eq}(\tau; p_\perp, w)}{\tau_{\rm rel}(\tau)} \tag{5}$$

with the solution [3, 4]

$$\begin{aligned} f(\tau; p_\perp, w) &= D(\tau,\tau_0) f_0(p_\perp, w) \\ &+ \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm rel}(\tau')} D(\tau,\tau')\, f_{\rm eq}(\tau'; p_\perp, w) \end{aligned} \tag{6}$$

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