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The right choice of moment for anisotropic fluid dynamics

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Abstract

We study anisotropic fluid dynamics derived from the Boltzmann equation based on a particular choice for the anisotropic distribution function within a boost-invariant expansion of the fluid in one spatial dimension. In order to close the conservation equations we need to choose an additional moment of the Boltzmann equation. We discuss the influence of this choice of closure on the time evolution of fluid-dynamical variables and search for the best agreement to the solution of the Boltzmann equation in the relaxation-time approximation.

1. Introduction

The basic axioms of fluid dynamics are the conservation laws of particle number and energy-momentum,

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0, \tag{1}$$

where N^μ is the particle four-current and $T^{\mu\nu}$ is the energy-momentum tensor. These are decomposed with respect to the normalized fluid four-velocity u^μ and the projection orthogonal to it $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, where $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the metric of space-time, as

$$N^\mu = n_0 u^\mu + V^\mu, \quad T^{\mu\nu} = e_0 u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}, \tag{2}$$

where $2a^{(\mu}b^{\nu)} \equiv a^\mu b^\nu + a^\nu b^\mu$. From these decompositions one can identify the following 14 quantities: The particle density $n_0 = N^\mu u_\mu$, energy density $e_0 = T^{\mu\nu}u_\mu u_\nu$, and total pressure $P_0 + \Pi = -\frac{1}{3}T^{\mu\nu}\Delta_{\mu\nu}$, being the sum of $P_0(n_0, e_0)$ and the bulk viscous pressure Π. Furthermore, the particle diffusion current $V^\mu = \Delta^\mu_\alpha N^\alpha$, the energy diffusion current $W^\mu = \Delta^\mu_\alpha T^{\alpha\beta}u_\beta$, and the shear-stress tensor $\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}T^{\alpha\beta}$, where $\Delta^{\mu\nu}_{\alpha\beta} = \frac{1}{2}\left(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$.

A sufficiently simple microscopic theory, where different assumptions for the derivation of the fluid-dynamical equations can be explicitly carried out and tested, is provided by the Boltzmann equation [1, 2]

$$k^\mu \partial_\mu f_{\mathbf{k}} = C\left[f_{\mathbf{k}}\right], \tag{3}$$

where $f_{\mathbf{k}} = f(x, k)$ is the single-particle distribution function, k^μ is the four-momentum of particles, and $C\left[f_{\mathbf{k}}\right]$ is the collision integral. A conventional way to derive fluid dynamics is to assume that the system is sufficiently close to local thermal equilibrium, characterized by a single-particle distribution function

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$f_{0\mathbf{k}}(\alpha_0,\beta_0)$, so that one can write $f_\mathbf{k} = f_{0\mathbf{k}} + \delta f_\mathbf{k}$, with $|\delta f_\mathbf{k}| \ll f_\mathbf{k}$, and then express $\delta f_\mathbf{k}$ in terms of only a few additional macroscopic quantities, e.g., as in Eq. (2).

The inverse temperature $\beta_0 = 1/T$ and chemical potential over temperature $\alpha_0 = \mu/T$, which appear as parameters in $f_{0\mathbf{k}}$, are usually defined through the so-called Landau matching conditions, i.e., by requiring that the particle density $n \equiv N^\mu u_\mu$ and energy density $e \equiv T^{\mu\nu} u_\mu u_\nu$ in the frame where the 4-velocity $u^\mu = (1,0,0,0)$ are identical to those of the local equilibrium state, $n = n_0(\alpha_0,\beta_0)$ and $e = e_0(\alpha_0,\beta_0)$. The Boltzmann equation can then be used to write down the equations of motion for the dissipative quantities appearing in Eq. (2). For details of the expansion and the derivation see e.g., Refs. [1, 2, 3].

Anisotropic fluid dynamics is based on a similar idea, but now the expansion is performed around a more general anisotropic distribution function $\hat{f}_{0\mathbf{k}}(\alpha_0,\beta_0,\xi)$, as $f_\mathbf{k} = \hat{f}_{0\mathbf{k}} + \delta\hat{f}_\mathbf{k}$, instead of an isotropic equilibrium state. Part of the possible deviations from the equilibrium distribution function $f_{0\mathbf{k}}$ can then be embedded into $\hat{f}_{0\mathbf{k}}$ [4, 6, 5, 7, 8]. If the momentum anisotropy is large, this can lead to a much faster convergence of the expansion. The degree of anisotropy in $\hat{f}_{0\mathbf{k}}$ is controlled by the new parameter ξ, while the direction of the anisotropy can be specified by a new spacelike 4-vector l^μ orthogonal to the four-velocity, $u^\mu l_\mu = 0$, and normalized to $l^\mu l_\mu = -1$. The additional parameter ξ needs to be determined by another matching condition in addition to the usual Landau matching conditions.

Now, N^μ and $T^{\mu\nu}$ need to be decomposed with respect to both four-vectors u^μ and l^μ as well as the two-space projector $\Xi^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu + l^\mu l^\nu$ orthogonal to both four-vectors [9]. This reads as

$$N^\mu = n\, u^\mu + n_l\, l^\mu + V_\perp^\mu, \tag{4}$$

$$T^{\mu\nu} = e\, u^\mu u^\nu + 2\, M\, u^{(\mu}\, l^{\nu)} + P_l\, l^\mu l^\nu - P_\perp\, \Xi^{\mu\nu} + 2\, W_{\perp u}^{(\mu} u^{\nu)} + 2\, W_{\perp l}^{(\mu} l^{\nu)} + \pi_\perp^{\mu\nu}, \tag{5}$$

where compared to Eq. (2) the particle diffusion current V^μ splits into two parts: The diffusion into the direction of l^μ given by $n_l = -N^\mu l_\mu$ and the part of the diffusion orthogonal to l^μ given by $V_\perp^\mu = \Xi^\mu_\alpha N^\alpha$. Similarly, the isotropic pressure splits into longitudinal and transverse parts $P = \frac{1}{3}(P_l + 2P_\perp)$, where $P_l = T^{\mu\nu} l_\mu l_\nu$ and $P_\perp = -\frac{1}{2}\, T^{\mu\nu}\Xi_{\mu\nu}$, and thus the shear-stress tensor from the second equation (2) can be written as

$$\pi^{\mu\nu} = \pi_\perp^{\mu\nu} + 2\, W_{\perp l}^{(\mu} l^{\nu)} + \frac{1}{3}\,(P_l - P_\perp)\,(2\, l^\mu l^\nu + \Xi^{\mu\nu})\,, \tag{6}$$

where $W_{\perp l}^\mu = -\Xi^\mu_\alpha T^{\alpha\beta} l_\beta$ and $\pi_\perp^{\mu\nu} = \Xi^{\mu\nu}_{\alpha\beta} T^{\alpha\beta}$, with $\Xi^{\mu\nu}_{\alpha\beta} = \frac{1}{2}\left(\Xi^\mu_\alpha \Xi^\nu_\beta + \Xi^\mu_\beta \Xi^\nu_\alpha\right) - \frac{1}{2}\Xi^{\mu\nu}\Xi_{\alpha\beta}$. Similarly to conventional fluid dynamics, the equations of motion for the anisotropic dissipative quantities can be also obtained from the Boltzmann equation, see e.g. Ref. [10].

2. Boost-invariant expansion

As a simple example to illustrate the importance of choosing the right matching, we utilize the anisotropic distribution function introduced by Romatschke and Strickland (RS) [11],

$$\hat{f}_{0\mathbf{k}} \equiv \hat{f}_{RS} = \left[\exp\left(\beta_0 \sqrt{E_{\mathbf{k}u}^2 + \xi E_{\mathbf{k}l}^2} - \alpha_0\right) + a\right]^{-1}, \tag{7}$$

where $E_{\mathbf{k}u} \equiv u^\mu k_\mu$ and $E_{\mathbf{k}l} \equiv -l^\mu k_\mu$. Note that when $\xi = 0$, this reduces to an equilibrium distribution function. We then solve anisotropic fluid dynamics in a simple 0+1 dimensional boost-invariant geometry, and compare the solution to the exact solution of the Boltzmann equation in the relaxation-time approximation (RTA) [12]. A general moment of the distribution function (7) can be written as

$$\hat{I}_{nrq}^{RS} = \frac{(-1)^q}{(2q)!!} \int dK\, E_{\mathbf{k}u}^{n-r-2q} E_{\mathbf{k}l}^{r} \left(\Xi^{\mu\nu} k_\mu k_\nu\right)^q\, \hat{f}_{RS}\;, \tag{8}$$

hence for example $\hat{n} = \hat{I}_{100}^{RS}$, $\hat{e} = \hat{I}_{200}^{RS}$, and $\hat{P}_l = \hat{I}_{220}^{RS}$, etc. Now, imposing the Landau matching conditions $\hat{n}(\alpha_{RS},\beta_{RS},\xi) = n_0(\alpha_0,\beta_0)$ and $\hat{e}(\alpha_{RS},\beta_{RS},\xi) = e_0(\alpha_0,\beta_0)$ we determine two parameters of the anisotropic distribution in terms of a fictitious equilibrium state.