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The thermalization of soft modes in non-expanding isotropic quark gluon plasmas

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Abstract

We discuss the role of elastic and inelastic collisions and their interplay in the thermalization of the quark–gluon plasma. We consider a simplified situation of a static plasma, spatially uniform and isotropic in momentum space. We focus on the small momentum region, which equilibrates first, and on a short time scale. We obtain a simple kinetic equation that allows for an analytic description of the most important regimes. The present analysis suggests that the formation of a Bose condensate, expected when only elastic collisions are present, is strongly hindered by the inelastic, radiative, processes.

Keywords: Perturbative QCD; Heavy ion collisions; Thermalization; Bose–Einstein condensation

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1. Introduction

Understanding how the gluons that are freed in the early stage of a heavy ion collision locally equilibrate and exhibit collective fluid behavior observed in experiments remains an important and challenging problem, with many interesting facets and open issues (see Refs. [1–3] for reviews). Within weak coupling approaches, to which the present discussion is limited, much

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of the relevant physical processes have been identified. These involve various plasma instabilities and their potential role in isotropizing the momentum distribution, the elastic and inelastic scatterings, and among the latter the importance of soft radiation [4,5], the role of the longitudinal expansion, etc. There exist detailed [4], and very detailed [6], parametric analyses at weak coupling of these various physical processes. Extended numerical simulations using statistical classical field theory point out the existence of regimes dominated by non-thermal fixed points, with characteristic scaling behavior (see [7] for a representative example). Additional insight is provided by kinetic theory, which focuses on the direct interactions between modes or quasiparticles [8,5,9]. From some of these works, evidence is emerging that thermalization can indeed be achieved among weakly coupled gluons on reasonably short time scales (see e.g. [10,11]).

However a fully coherent picture is still lacking, and many issues remain to be clarified. Furthermore all approaches have limitations. For instance, numerical simulations have difficulties to handle very long wavelength modes, and they do not always offer the physical insight that one is looking for. As for the parametric estimates, they are blind to the possible presence of large numerical factors (and there are such factors) that can obscure the picture; one may be led for instance to expect the existence of regimes that the full solution does not reveal, because in practice, scales are not as well separated as the parametric analysis would suggest. Given the overall complexity of the problem, we feel therefore that there is a need for developing an understanding based on simple (differential) equations that we can control (semi) analytically. Our goal is certainly not to resolve all the pending issues, but to identify robust, generic qualitative behaviors and understand the basic mechanisms and the factors that control the important time scales. This requires simplifications, in the choice of the system to be studied, and in the choice of an appropriate theoretical framework that may allow for analytical insight. The simplifications concerning the system to be studied are standard, and they can be alleviated with moderate effort: thus, in this paper, we shall study a uniform, non-expanding system, which furthermore is isotropic in momentum space. Moreover we shall focus on small momentum modes. Relaxing these assumptions is possible, as just said, and will be the subject of future publications. As for the choice of the framework, we choose to work within kinetic theory, to which we now turn.

We shall use a kinetic equation of the generic form

$$\frac{\partial f(t,\boldsymbol{p})}{\partial t} = C_{\rm el}[f] + C_{\rm inel}[f], \tag{1}$$

where the collision integral is conveniently split into two contributions: one corresponding to elastic, number conserving, scattering; the other referring to inelastic, number changing, processes. Such an equation has been used, implicitly or explicitly in many discussions of the thermalization of the quark–gluon plasma (see e.g. [4,12,13]).

The present work builds on our previous works on the subject [14] where only elastic scatterings were taken into account, i.e., $C_{\rm inel}[f]$ was set to zero. Because the dominant scattering among gluons occur at small angle, we can reduce the Boltzmann equation to a (much simpler) Fokker–Planck equation, in which the collision integral is written as the divergence of a current (in momentum space). The small scattering angle approximation has been checked against numerical solutions of the Boltzmann equation, and no major qualitative differences could be observed [9,15]. The Fokker–Planck equation provides a simple visualization of the physics in terms of competing currents of particles in momentum space. A particular striking consequence of this analysis is the prediction that, for typical initial conditions, gluons could undergo Bose–Einstein condensation (BEC), as first suggested in [16]. Of course inelastic scattering will eventually prevent a true condensate to be present in the equilibrium state, but a priori this does

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