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Half-BPS Wilson loop and AdS_2/CFT_1

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Abstract

We study correlation functions of local operator insertions on the 1/2-BPS Wilson line in $\mathcal{N} = 4$ super Yang–Mills theory. These correlation functions are constrained by the 1d superconformal symmetry preserved by the 1/2-BPS Wilson line and define a defect CFT_1 living on the line. At strong coupling, a set of elementary operator insertions with protected scaling dimensions correspond to fluctuations of the dual fundamental string in $AdS_5 \times S^5$ ending on the line at the boundary and can be thought of as light fields propagating on the AdS_2 worldsheet. We use AdS/CFT techniques to compute the tree-level AdS_2 Witten diagrams describing the strong coupling limit of the four-point functions of the dual operator insertions. Using the OPE, we also extract the leading strong coupling corrections to the anomalous dimensions of the "two-particle" operators built out of elementary excitations. In the case of the circular Wilson loop, we match our results for the 4-point functions of a special type of scalar insertions to the prediction of localization to 2d Yang–Mills theory.

1. Introduction

In the $\mathcal{N} = 4$ supersymmetric Yang–Mills theory, it is natural to consider Wilson loop operators that include couplings to the six scalars Φ^I in the theory [1,2]

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$$W = \mathrm{tr} P e^{\oint dt \left(i\dot{x}^\mu A_\mu + |\dot{x}| \theta^I \Phi^I \right)} , \tag{1.1}$$

where $x^\mu(t)$ is a closed loop, and $\theta^I(t)$ is a unit 6-vector. For generic contour and scalar couplings, these operators are only locally supersymmetric, but special choices of x^μ and θ^I lead to Wilson loops preserving various fractions of the superconformal symmetry of the $\mathcal{N}=4$ SYM theory [3,4]. The most supersymmetric operator is obtained by taking the contour to be an infinite straight line (or circle), and θ^I a constant 6-vector, corresponding to a fixed direction in the scalar space: in this case the Wilson loop is 1/2-BPS, i.e. it preserves 16 of the 32 supercharges of the superconformal group $PSU(2,2|4)$. Making the choice $\theta^I \Phi^I = \Phi^6$, this 1/2-BPS straight line operator is given by

$$W = \mathrm{tr} P e^{\int dt \left(i A_t + \Phi^6 \right)} \tag{1.2}$$

where we have identified the Euclidean time $x^0 = t \in (-\infty, \infty)$ to be the line that defines the operator.

In this paper we will be interested in the computation of correlation functions of local operators inserted along the straight Wilson line, defined as follows. Given some local operators $O_i(t_i)$ transforming in the adjoint representation of the gauge group, one can define the gauge invariant correlator [5]

$$\begin{aligned} &\langle\langle O_1(t_1) O_2(t_2) \cdots O_n(t_n) \rangle\rangle \\ &\quad \equiv \langle \mathrm{tr} P \big[O_1(t_1)\, e^{\int dt (i A_t + \Phi^6)}\, O_2(t_2)\, e^{\int dt (i A_t + \Phi^6)} \cdots\, O_n(t_n)\, e^{\int dt (i A_t + \Phi^6)} \big] \rangle \\ &\quad \equiv \langle \mathrm{tr} P \big[O_1(t_1) O_2(t_2) \cdots O_n(t_n) e^{\int dt (i A_t + \Phi^6)} \big] \rangle \, . \end{aligned} \tag{1.3}$$

The $SU(N)$ indices are contracted with the Wilson lines joining the various points, making this a gauge invariant observable. Since the expectation value of the straight Wilson line is trivial, this definition satisfies $\langle\langle 1 \rangle\rangle = \langle W \rangle = 1$. More generally, one should normalize the correlator on the right-hand side by the expectation value of the Wilson loop without insertions (this is relevant in the case of the 1/2-BPS circular loop, which has a non-trivial expectation value [6–8]). Note that, since operator insertions are equivalent to deformations of the Wilson line [5,9], the complete knowledge of the correlators (1.3) would, at least in principle, allow to compute the expectation value of general Wilson loops which are deformations of the line or circle.

To understand the structure of the correlators (1.3), it is useful to recall the symmetries preserved by the 1/2-BPS Wilson line. First, it is clear that it preserves an $SO(5)$ subgroup of the $SO(6)_R$ R-symmetry that rotates the 5 scalars Φ^a, $a = 1, \ldots, 5$ that do not couple to the Wilson loop. In addition, it preserves an $SO(2,1) \times SO(3)$ subgroup of the 4d conformal group $SO(2,4)$, where the $SO(3)$ corresponds to rotations around the line, and the generators of $SO(2,1)$ correspond to dilatations, translation and special conformal transformation along the line. This $SO(2,1)$ is the $d=1$ conformal group. Together with the 16 supercharges preserved by the loop, the symmetries of 1/2-BPS Wilson lines form the $d=1$, $\mathcal{N}=8$ superconformal group $OSp(4^*|4)$.

It follows that operator insertions along the Wilson line are classified by their representations under the $OSp(4^*|4)$ symmetry. In particular, they are labeled by their scaling dimension Δ, corresponding to a representation of $SO(2,1)$, and by a representation of the "internal" (from the point of view of the line) symmetry group $SO(3) \times SO(5)$. The set of correlators (1.3) are then constrained by the $d=1$ conformal symmetry in a way analogous to higher dimensional CFTs. They can be interpreted as characterizing a defect CFT_1 living on the Wilson line [5,10,9]. This CFT_1 should then be fully determined by its spectrum of scaling dimensions and OPE