Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 920 (2017) 368–384
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Revisiting the dilatation operator of the Wilson–Fisher fixed point

Pedro Liendo

DESY Hamburg, Theory Group, Notkestrasse 85, D-22607 Hamburg, Germany

Received 3 March 2017; received in revised form 6 April 2017; accepted 27 April 2017
Available online 3 May 2017
Editor: Leonardo Rastelli

Abstract

We revisit the order-ε dilatation operator of the Wilson–Fisher fixed point obtained by Kehrein, Pismak, and Wegner in light of recent results in conformal field theory. Our approach is algebraic and based only on symmetry principles. The starting point of our analysis is that the first correction to the dilatation operator is a conformal invariant, which implies that its form is fixed up to an infinite set of coefficients associated with the scaling dimensions of higher-spin currents. These coefficients can be fixed using well-known perturbative results, however, they were recently re-obtained using CFT arguments without relying on perturbation theory. Our analysis then implies that all order-ε scaling dimensions of the Wilson–Fisher fixed point can be fixed by symmetry.

1. Introduction

The Wilson–Fisher fixed point is one of the paradigmatic examples of conformal field theory (CFT). It can be obtained from a free scalar in $d = 4 - \varepsilon$ dimensions by adding a quartic interaction and flowing to the IR. The usual approach is to consider a perturbative expansion in the parameter ε, with the hope that setting $\varepsilon = 1$ will give an approximate description of three-dimensional physics [1]. The continuation of the Wilson–Fisher fixed point to $d = 3$ belongs to the same universality class of the critical Ising model, and indeed, critical exponents obtained using the ε-expansion are in good agreement with results obtained by other methods [2].

E-mail address: pedro.liendo@desy.de.

http://dx.doi.org/10.1016/j.nuclphysb.2017.04.020
0550-3213/

The main motivation behind this work is the bootstrap approach to CFTs [3–5], which attempts to constrain the dynamics of a theory using only the conformal algebra. This approach has experienced a revival thanks to the pioneering numerical work of [6], that has motivated many recent papers on the bootstrap, but also influenced research on related areas, like mathematical physics, supersymmetric field theory, and holography. Of all these recent developments, perhaps the most impressive is the high-precision estimates of critical exponents in the three-dimensional Ising model [7–11].

The new developments have also led to revisit the ε-expansion through the lens of the bootstrap.[1] This approach was first considered in the context of boundary CFT, where an attempt was made to recover the ε-expansion starting from crossing symmetry [14]. A different strategy not based in crossing was presented in [15] (see also [16–23]), where the constraints imposed by multiplet recombination turned out to be powerful enough to fix the order ε anomalous dimensions of operators of the form ϕ^n. Three more techniques include using the singularity structure of conformal blocks [24,25], unitarity methods [26], and the Mellin bootstrap [27–29], where corrections up to order ε^3 have been obtained.

Another approach based on the equations of motion and similar in spirit to [15] was used in [30,31] (see also [32]), where the leading order anomalous dimensions of higher-spin currents were obtained. In particular, the order-ε anomalous dimensions vanish, which will play a crucial role in our analysis. The same result was obtained without recourse to a Lagrangian description in [33,34] using the large-spin limit of crossing symmetry [35,36], and in [23] using a generalization of the method of [15].

The purpose of this paper is to study the complete order ε dilatation operator of the Wilson–Fisher fixed point, which allows to efficiently calculate the anomalous dimension of any operator in the theory. This result is not new, it was obtained by a careful analysis of Feynman diagrams by Kehrein, Pismak, and Wegner in [37]. They packaged their result in an elegant formula and checked a posteriori that their expression was invariant under the conformal algebra. Here we will reverse the logic, and consider invariance of the dilatation operator as our starting point, as shown below, this drastically simplifies the calculation. This approach was successfully used by Beisert to obtain the complete one-loop dilatation operator of $\mathcal{N} = 4$ SYM [38,39]. The technique however is quite general and not restricted to superconformal theories. It turns out that the few coefficients that cannot be fixed by Beisert's approach, have been recently obtained using the aforementioned bootstrap-based arguments [33,34,23]. Thus, the dilatation operator of the Wilson–Fisher fixed point can be completely fixed by symmetry, bypassing Feynman diagram calculations.

2. The order ε dilatation operator

Let us consider a scalar field in d dimensions with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4 \,. \tag{2.1}$$

It is well known that this theory has an interacting IR fixed point for dimensions $2 < d < 4$, and by considering the theory in $d = 4 - \varepsilon$ dimensions it can be studied perturbatively. The β function at order ε is

[1] For interesting related work that considers a large-charge expansion of CFTs see [12,13].