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Nuclear Physics B 923 (2017) 144–167

NUCLEAR PHYSICS B

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Dimensional flow and fuzziness in quantum gravity: Emergence of stochastic spacetime

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Received 5 June 2017; accepted 26 July 2017
Available online 2 August 2017
Editor: Stephan Stieberger

Abstract

We show that the uncertainty in distance and time measurements found by the heuristic combination of quantum mechanics and general relativity is reproduced in a purely classical and flat multi-fractal spacetime whose geometry changes with the probed scale (dimensional flow) and has non-zero imaginary dimension, corresponding to a discrete scale invariance at short distances. Thus, dimensional flow can manifest itself as an intrinsic measurement uncertainty and, conversely, measurement-uncertainty estimates are generally valid because they rely on this universal property of quantum geometries. These general results affect multi-fractional theories, a recent proposal related to quantum gravity, in two ways: they can fix two parameters previously left free (in particular, the value of the spacetime dimension at short scales) and point towards a reinterpretation of the ultraviolet structure of geometry as a stochastic foam or fuzziness. This is also confirmed by a correspondence we establish between Nottale scale relativity and the stochastic geometry of multi-fractional models.

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http://dx.doi.org/10.1016/j.nuclphysb.2017.07.016
0550-3213/

1. Introduction

After many years of research, we are not yet close to an acknowledged unique quantum theory of gravity, partly because of the lack of experimental guidance. The mathematical and conceptual challenges raised by the attempt of combining quantum-mechanical and general-relativistic principles produced plenty of different approaches to the problem of quantum gravity (QG) [1–3]. Among them, we count string theory [4], the tripod of group field theory, loop quantum gravity and spin foams [5–8], causal dynamical triangulation [9], causal sets [10], asymptotically safe gravity [11–13], non-commutative spacetimes [14,15] and non-local quantum gravity [16–19], just to mention some of the most popular models available in the literature. Over the last twenty years, this considerable theoretical effort has started both to figure out phenomenological predictions that could be tested with the presently-achievable levels of experimental sensitivity and to gradually focus on few results that seem independent of the specific quantum-gravity framework adopted [20]. In fact, even if there are great differences between inequivalent approaches, some common features have been noticed. One of the most recurrent findings in the field is *dimensional flow* (or dimensional running), i.e., a change of spacetime dimension with the scale of the observer. In almost all quantum-gravity models, the dimensionality of spacetime exhibits a dependence on the scale, changing (or "flowing") from the topological dimension D in the infrared (IR) to a different value in the ultraviolet (UV). There can be more than a single relevant scale and, thus, the dimension can change many times before reaching its far-UV value at a scale that is often identified with (or recognized as) the Planck length $\ell_{\rm Pl} = (G\hbar/c^3)^{1/(D-2)}$. Sometimes, the concept of dimension does not even survive deep into these UV scales and it dissolves into some highly non-smooth structure (for instance, multi-fractal, discrete, or combinatorial). All known quantum gravities are *multi-scale* by definition because they all have an anomalous scaling of the dimension [21–23] (see [24,25] for a scan of the literature and more and newer references). A recent strategy for easily realizing the running of the dimension has been followed by *multi-fractional theories*, comprehensively reviewed in [24]. In these models, the basic ingredient implementing dimensional flow is a non-trivial factorizable integration measure

$$dq^0(x^0)dq^1(x^1)\cdots dq^{D-1}(x^{D-1}) = \frac{dq^0}{dx^0}dx^0\frac{dq^1}{dx^1}dx^1\cdots\frac{dq^{D-1}}{dx^{D-1}}dx^{D-1}\,.$$

The profiles $q^\mu(x^\mu)$ are determined uniquely and solely by requiring to reach the IR limit as an asymptote [26]. An approximation of the full measure, which will be of interest here, is the so-called binomial space-isotropic profile

$$q^\mu(x^\mu) \simeq (x^\mu - \bar{x}^\mu) + \frac{\ell_*}{\alpha_\mu}\left|\frac{x^\mu - \bar{x}^\mu}{\ell_*}\right|^{\alpha_\mu}, \qquad \alpha_\mu = \alpha_0, \alpha. \tag{1}$$

Here there is no summation over the index $\mu = 0, \ldots, D-1$. The fractional exponents $0 < \alpha_0, \alpha < 1$ are directly related to both the spectral and the Hausdorff dimensions ($d_{\rm S}$, $d_{\rm H}$) at very short distances $\ell \lesssim \ell_*$; if $\alpha_0 = \alpha$, then $d_{\rm S} \simeq D\alpha \simeq d_{\rm H}$ in the UV for the theories considered here (with fractional or q-derivatives [24]). In the above measure, we are assuming spatial isotropy (same α for all space directions) and the existence of only one characteristic length ℓ_*. These approximations can be relaxed without difficulty but, since the full exact form of the measure $q^\mu(x^\mu)$ is not needed here,[1] for the sake of our argument we will limit our attention

[1] The binomial measure is the approximation of a measure with many scales smaller than ℓ_*, all of which are effectively "screened" by ℓ_* and that do not appear in the phenomenology for all practical purposes [26].